FLA (Fall 2022) – Assignment 1

Name: ______________ Dept: ______________

Grade: ______________ ID: ______________

Due: Oct. 18, 2022

Problem 1

Provide DFAs and REs of the following languages. In all parts, the alphabet $\Sigma = \{0, 1\}$ and $|v|_\omega$ means the number of substring v occurrences in string ω.

a. $\{\omega \mid |01|_\omega \geq 2 \}$

b. $\{\omega \mid \omega$ contains at least one 1 and two 0 $\}$

c. $\{\omega \mid \omega$ has odd length and ends with 1, or, has even length and ends with 0 $\}$

d. $\{\omega \mid |01|_\omega \bmod 2 \ \equiv\ 0 \}$ (DFA only)

Solution.

Problem 2

Let $R = (\mathbf{a} + \mathbf{b})^* \mathbf{a} (\mathbf{b} + \mathbf{c})^* \mathbf{b} (\mathbf{a} + \mathbf{c})^*$.

a. Convert R to an ϵ-NFA.

b. Convert the ϵ-NFA to a DFA by subset construction.

Solution.

Problem 3

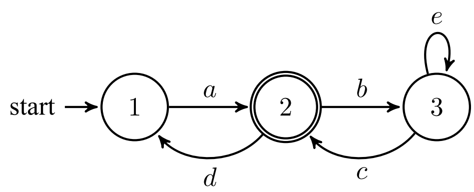
Give a DFA as figure below, please give the regular expression for the following R_{ij}^{k}, and try to simplify the expressions as much as possible.

a. All the REs R_{ij}^{0}

b. All the REs R_{ij}^{1}

c. All the REs R_{ij}^{2}

d. The RE for this DFA



Solution.

Problem 4

Prove that the following languages are not regular. You may use the pumping lemma and the closure properties of the class of regular languages.

a. $\{\omega\omega \mid \omega \in \{0,\ 1\}^* \}$

b. $\{\omega \mid\ |0|_\omega \geq |1|_\omega,\ \omega \in \{0,\ 1\}^* \}$

c. $\{\omega \mid |0|_\omega + |1|_\omega = n^2,\ n \in N \}$

d. $\{0^x \mid$ x is a prime number $\}$

Proof.

Problem 5

Let a ”restricted DFA” be any DFA that contains exactly one accepting state. Are restricted DFAs strictly less powerful than DFAs in terms of language expressing? If so, give an example regular language and prove that it cannot be decided by any restricted DFA. If not, prove the equivalence between DFAs and restricted DFAs.

Proof.

Problem 6

Let's define a new binary operator $\star$ for languages. Let $L_1, L_2 \in \Sigma^*$, where Σ is an alphabet. We define

$$L_1 \star L_2 = \{w | (w \in L_1 \vee w \in L_2) \wedge \neg(w \in L_1 \wedge w \in L_2)\}.$$

Is the class of regular languages closed under $\star$? If so, give a sound proof. If not, give an counterexample.

Problem 7

Prove or disprove the following statement:

a. Every regular language has a regular proper subset.

b. A, B are two languages over alphabet Σ. If neither A nor B is regular, then $A \cup B$ is also not regular.

c. If A and B are not regular languages and C is a language such that $A \subseteq C \subseteq B$, then C is not regular.

d. An infinite regular language L can be split into two infinite disjoint, nonempty, regular subsets L_1, L_2. (**Hint: use the pumping lemma.**)

Solution.

Problem 8 (Bonus.)

a. For a language L, let $A(L)$ be the language $\{wv | \exists v \in \Sigma . vw \in L\}$. Show that if L is regular, so is $A(L)$.

b. For a language L, let $B(L)$ be the language $\{wv | \exists v \in \Sigma^* . vw \in L\}$. Show that if L is regular, so is $B(L)$.

Solution.