FLA (Fall 2022) – Assignment 2

Name: ______________ Dept: ______________

Grade: ______________ ID: ______________

Due: Nov. 15, 2022

Problem 1

Give context free grammars that generate the following languages, and give a brief description of the functionality of each variable in your grammars (in natural language).

a. $\{w \in \{a,b\}^* \mid w = w^R\}$

b. $\{w \in \{a,b\}^* \mid |b|_w = 2|a|_w\}$, where $|x|_w$ denotes the number of occurrences of x in string w.

c. $\{a^i b^j c^k \mid i,j,k \geq 0,\ and\ i+j=k\}$

Problem 2

Consider the following context free grammar: G = ({ S }, { x, +, (,) }, P, S), where P consists of:

$$S \to S + S \mid (S) \mid x$$

Is grammar G ambiguous? If it is, give an example string that has more than one parse tree according to G, and give an unambiguous grammar that generates the same language. If not, prove your conclusion.

Problem 3

Consider the following context free grammar: G = ({ S, B }, { 0, 1 }, P, S), where P consists of:

$$S \rightarrow BSA \mid A$$

$$A \rightarrow 0A \mid \epsilon$$

$$B \rightarrow B10 \mid \epsilon$$

a. For the string 1010000, give its parse tree and rightmost derivation according to G.

b. Provide a nondeterministic PDA P that accepts the language $L(G)$ by empty stack.

Problem 4

Begin with the grammer:

$$S \to BSA \mid A$$

$$A \to aA \mid \epsilon$$

$$B \to Bba \mid \epsilon$$

1. Eliminate ϵ-productions.

2. Eliminate any unit productions in the resulting grammar of (1.).

3. Put the resulting grammar of (2.) into Chomsky normal form.

Problem 5

Given grammer G:

$$S \to aSb \mid bSa \mid \epsilon$$

Please use CYK algorithm to decide whether string $aabbabb$ belongs to $L(G)$.

Problem 6

Prove that each of these languages is not context free.

a. $L = \left\{ 0^{i^2} \mid i \geq 0 \right\}$.

b. $L = \{ 0^i 1^j \mid i^2 \geq j \}$.

c. $L = \{ \omega \in \{0,\ 1,\ 2\}^* \mid \omega$ has equal number of 0's, 1's and 2's $\}$.

d. **(Bonus)** $L = \{ \omega\omega\omega \mid \omega \in \{0,\ 1\}^* \}$.

Problem 7

For any context-free language L and any regular language R, answer each of the following statements **True** or **False**. If your answer is **True**, give an explanation. If your answer is **False**, give a counterexample.

a. $L - R$ is context-free.

b. $R - L$ is context-free.

c. $S(L) = \{w | \exists v \in \Sigma^*. vw \in L\}$. $S(L)$ is context-free.

d. **(Bonus)** $H(L) = \{w | \exists v \in \Sigma^*. vw \in L \wedge |v| = |w|\}$. $H(L)$ is context-free. **(Hint: intersection with a regular language.)**

Problem 8

We define an operation $\bowtie$ for language L and R to be

$$L \bowtie R = \{w \mid w = x_1y_1x_2y_2 \cdots x_ny_n \text{ for some } n, \text{ where } x_1x_2 \cdots x_n \in L \text{ and } y_1y_2 \cdots y_n \in R, \text{ each } x_i, y_i \in \Sigma^*\}$$

a. Show that if L is context-free and R is regular, then $L \bowtie R$ is context-free.

b. Show that the class of CFL is not closed under $\bowtie$ operation.