FLA (Fall 2022) – Assignment 3

Name: ________________ Dept: ________________

Grade: ________________ ID: ________________

Due: 17 Dec 2022

Problem 1

Consider the (deterministic) Turing machine M given by

$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, \{q_4\})$$

which has exactly six transitions defined in it, as described below.

$$\delta(q_0, a) = (q_1, B, L) \quad \delta(q_0, b) = (q_2, B, L) \quad \delta(q_0, B) = (q_4, B, L)$$

$$\delta(q_1, B) = (q_3, a, R) \quad \delta(q_2, B) = (q_3, b, R) \quad \delta(q_3, B) = (q_0, B, R)$$

Please answer the following questions:

a. Specify the execution trace of M on the input string $abab$.

b. What is the language accepted by this Turing machine?

c. Describe the function of this Turing machine in natural language.

Solution.

Problem 2

a. Design a single-taped TM M_1 to perform ADD operation. The input string is in the form $u\#v$, where $u, v \in \{1\}^*$. The output string $w \in \{1\}^*$ with $|w| = |u| + |v|$. You can assume the input alphabet is $\Sigma = \{1, \#\}$.

b. Design a single-taped TM M_2 to decide the language $L = \{wcw | w \in \{a, b\}^*\}$. You can assume the input alphabet is $\Sigma = \{a, b, c\}$.

Solution.

Problem 3

We know that a turing machine can have its own output string. Now denote the output string of turing machine M on input i as $f_M(i)$. Suppose there exists a turing machine M (which will always halt on any input) and a language L that satisfies $\forall i, f_M(i) \in L$, and $\forall w \in L,\ \exists i,\ s.t.\ f_M(i) = w$.

a. Prove that L is recursively enumerable.

b. For bijection $f : N \to \Sigma^*$, define a total order $\preccurlyeq$ on Σ^* as $m \leq n \Leftrightarrow f(m) \preccurlyeq f(n)$. If $f_M(i)$ is monotonically increasing (i.e. $\forall u \forall v,\ u \preccurlyeq v \Rightarrow f_M(u) \preccurlyeq f_M(v)$), prove that L is recursive.

Proof.

Problem 4

a. Recall that a Push-down Automaton is a finite automaton that can manipulate a stack. Now define ”Pop-up Automaton” as any finite automaton that can manipulate a queue, just like the way a PDA manipulates its stack. Show informally that PUA can simulate a Turing Machine.

b. Now define ”Restricted Turing Machine” as a Turing Machine whose read-write head cannot move leftward (i.e. it can only stay or move rightward). Show informally that all languages accepted by RTMs are regular.

Solution.

Problem 5

Answer the following statements **True** or **False**. If your answer is **True**, give an explanation. If your answer is **False**, give a counterexample. Please avoid using Rice's Theorem.

a. $\mathcal{P}$ is closed under **Kleene Closure**. So is $\mathcal{NP}$.

b. $\mathcal{P}$ is closed under **Complement**.

c. $L = \{ \langle D \rangle \mid D \text{ is a DFA and } L(D) \text{ is finite } \}$ is decidable.

d. $L = \{ \langle M \rangle \mid M \text{ is a turing machine and } L(M) \text{ is finite } \}$ is decidable

Solution.

Problem 6

Consider two problems SHORT-k-PATH and LONG-k-PATH. Both take input (G, u, v, k), where G is a graph, u and v are vertices, and k is a positive integer. SHORT-k-PATH decides whether there is any path from u to v of length at most k in G, while LONG-k-PATH decides whether there is any path of length at least k. Note that paths are not allowed to visit the same vertex repeatedly. Show that SHORT-k-PATH is in $\mathcal{P}$, but LONG-k-PATH is $\mathcal{NP}$-complete.
(**Hint**: We have already shown some $\mathcal{NP}$-complete problems in the slides.)

Solution.

Problem 7

Define

$$L = \left\{ w \,\middle|\, (\exists x . x \in \text{HALT} \wedge w = 0x) \vee \big(\exists y . y \in \overline{\text{HALT}} \wedge w = 1y\big) \right\},$$

where

$$\text{HALT} = \{ \langle M, w \rangle \mid M \text{ halts on input } w \} .$$

Prove that if neither L nor its complement is recursively enumerable.

Proof.