PTC (Fall 2018) – Assignment 3

Name: _____ Dept: _____

Grade: _____ ID: _____

Due: Dec. 20, 2018

Problem 1

Consider the (deterministic) Turing machine M given by

$$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, \{q_2\})$$

which has exactly four transitions defined in it, as described below.

1. $\delta(q_0, a) = (q_0, B, R)$
2. $\delta(q_0, b) = (q_1, B, R)$
3. $\delta(q_1, b) = (q_1, B, R)$
4. $\delta(q_1, B) = (q_2, B, R)$

Please answer the following questions:

a. Specify the execution trace of M on the input string abb.

b. Provide a regular expression for the language of the Turing machine.

c. Suppose we added the transition $\delta(q_1, a) = (q_0, B, R)$ to the above machine, provide a regular expression for the language of the resulting Turing machine.

Solution.

Problem 2

Please design TM′s to decide following languages:

a. $L_1 = \{1^m \times 1^n = 1^{mn} \mid m, n \in \mathbb{N}^+\}$ (e.g. $11 \times 111 = 111111 \in L_1$, but $1 \times 1 = 11 \notin L_1$)

b. $L_2 = \{ww \mid w \in \{a, b\}^*\}$

Solution.

Problem 3

A *useless state* in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a state in a Turing machine is useless. Formulate this problem as a language and show it is decidable or undecidable. (**Hint**: consider the language E_{TM})

Solution.

Problem 4

Show that the following questions are decidable:

a. The set L of codes for TM's M such that, when started with the blank tape will eventually write some nonblank symbol on its tape. (**Hint**: If M has m states, consider the first m transitions that it makes)

b. The set L of codes for TM's that never make a move left on any input.

c. The set L of pairs (M, w) such that TM M, started with input w, never scans any tape cell more than once.

Proof.

Problem 5

If a pushdown automaton has k stacks, we call it $k-$PDA. Clearly, $0-$PDA is NFA, $1-$PDA is PDA, and $1-$PDA is more powerful than $0-$PDA.

1. What is the difference between the express ability of 2-PDA and 1-PDA. Please clarify your argument. Prove the (un)equivalence.
2. How about 3-PDA and 2-PDA.

Solution.

Problem 6

Suppose we have an encoding of context-free grammars using some finite alphabet. Consider the following two languages:

1. $L_1 = \{(G, A, B) \mid G$ is a (coded) CFG, A and B are (coded) varibles of G, and the sets of terminal strings derived from A and B are the same$\}$.
2. $L_2 = \{(G_1, G_2) \mid G_1$ and G_2 are (coded) CFG′s, and $L(G_1) = L(G_2)\}$.

Answer the following questions:

a. Show that L_1 is polynomial-time reducible to L_2.

b. Show that L_2 is polynomial-time reducible to L_1.

Proof.

Problem 7

As classes of languages, $\mathcal{P}$ and $\mathcal{NP}$ each have certain closure properties. Prove or disprove that $\mathcal{P}$ and $\mathcal{NP}$ are closed under each of the following operations:

a. Union.

b. Concatenation.

c. Complementation.

Proof.