考试科目名称　算法设计与分析　（A 卷）

考试方式：　闭卷　考试日期　2008　年　01　月　13　日　教师　陈道蓄

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题号	一	二	三	四	五	六	七	八	九	十
分数	15	10	15	15	15	30				

得分 ＿＿　1、（本题满分 15 分）

For each pair of expressions (A, B) below, indicate whether A is O, Ω, Θ of B.

(1) $A=n^{1000}$, $B=2^n$　　(2) $A=(\lg n)^{12}$, $B=\sqrt{n}$　　(3) $A=\lg(n!)$, $B=n\lg n$

(4) $A=10^n$, $B=100^n$　　(5)$A=n^{\lg n}$, $B=(\lg n)^n$

得分 ＿＿　2、（本题满分 10 分）

Consider an open-address hash table with uniform hashing. Give upper bounds on the expected number of probes in an unsuccessful search and on the expected number of probes in a successful search when the load factor is 3/4 and when it is 7/8.

得分	

3、（本题满分 15 分）

Assume a undirected graph is represented in adjacency list form, answer problem (1) and (2-1) OR (2-2)

(1) Describe the strategy and the outline of implementation of Breadth-First Search for traversing a graph.

(2-1) A forest is a graph composed of zero or more disconnected trees. Design and analyze an algorithm that, given a graph G with n nodes, determines whether G is a forest in time $O(n)$.

(2-2) A bipartite graph is one whose vertex set can be partitioned into two sets A and B, such that each edge in the graph goes between a vertex in A and a vertex in B. (No edges between nodes in the same set are allowed). Design and analyze a $O(|E|+|V|)$ algorithm that takes an input graph and decides if the graph is bipartite. If the graph is bipartite, the algorithm should also produce the bipartition.

得分	

4、（本题满分 15 分）

Given two sorted sequence with m, n elements, respectively, designed and analyze an efficient algorithm (in time $O(\lg(m+n))$) to find the k-th element in the merge of the two sequences. (You may consider a divide-and-conquer algorithm)

得分	

5、（本题满分 15 分）

Given a set $A=\{s_1,s_2,...,\ s_n\}$, where s_i (for $i=1,2,\ldots,\ n$) is a natural number, and a nutural number S, determine whether there is a subset of A totaling exactly S. Design a dynamic programming algorithm for solving the problem. (using a two-dimension boolean table T, in which $T[i,j]$=**true** if and only if there is a subset of the first items of A totaling exactly j.)

得分	

6、（本题满分 30 分）

A boolean formula is in *disjunctive normal form* (DNF) if it consists of clauses of conjunctions (ANDs) joined together by disjunctions (ORs). For example, the formula (a ∧ b ∧ c) ∨ (b ∧ c) ∨ (a ∧ b ∧ c) is in disjunctive normal form. DNF-SAT is the problem that asks, given a boolean formula in disjunctive normal form, whether that formula is satisfiable.

(a) Prove that polynomial reduction is a transitive relation. (5 scores)

(b) Given a known NP-complete problem P, describe how to prove that a problem Q is an NP-complete problem as well. (5 scores)

(c) Show that DNF-SAT is in P. (10 scores)

(d) What is wrong with the following argument that P=NP? (10 scores)

Suppose we are given a boolean formula in conjunctive normal form with at most three literals per clause, and we want to know if it is satisfiable. We can use the distributive law to construct an equivalent formula in disjunctive normal form. For example,

(a ∨ b ∨ c) ∧ (a ∨ b) ≡ (a ∧ b) ∨ (b ∧ a) ∨ (c ∧ a) ∨ (c ∧ b)

Now we can use the answer to part (a) to determine, in polynomial time, whether the resulting DNF formula is satisfiable. We have just solved 3SAT in polynomial time! Since 3SAT is NP-hard, we must conclude that P=NP.