

Fast Power Allocation Algorithm for Cognitive Radio Networks

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Abstract—A fast algorithm is proposed to tackle the optimal power allocation problem for orthogonal frequency division multiplexing (OFDM)-based cognitive radio networks, where the key is to replace the Newton step with $O(N^3)$ complexity in the barrier method by a procedure with approximate linear complexity developed based on the structure of the optimization problem. Simulation results validate that our method can always work out the optimal solution, with complexity even lower than heuristic methods that can only produce suboptimal solutions.

Index Terms—Cognitive Radio, Convex Optimization, OFDM, and Resource Allocation.

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM)-based cognitive radio (CR) technology can improve the utilization efficiency of the radio spectrum unused by the licensed user (or primary user, PU) [1][2]. Power allocation is an important issue in OFDM-based cognitive radio networks [3]. Optimal power allocation, however, is generally intractable, and many heuristic algorithms have been proposed to achieve suboptimal solutions [4][5], with a significant performance gap with the optimal solution. In this Letter, we show that the optimal power allocation problem has a special optimization structure which can be exploited to reach the optimal solution with a low computational load.

II. SYSTEM MODEL

Consider the downlink of a CR system with OFDM modulation coexisting with a licensed system. Both of them share the same spectrum of bandwidth W . The total bandwidth is divided into N OFDM subchannels in the CR network. The bandwidth of the n th OFDM subchannel spans from $f_0 + (n-1)\frac{W}{N}$ to $f_0 + n\frac{W}{N}$, where f_0 is the starting frequency. There are M active PUs with unknown modulation manner in the licensed system. The spectrum of the m th PU spans from f_m to $f_m + W_m$, where W_m is the bandwidth of the m th PU. The interference to the m th PU introduced by the CR

user (also referred to as the secondary user, SU) must be kept below I_m . The power spectrum density (PSD) of the OFDM subcarrier is

$$\phi(f) = T \left(\frac{\sin \pi f T}{\pi f T} \right)^2, \quad (1)$$

where T is the OFDM symbol duration. The interference introduced to the m th PU by the n th OFDM subchannel with unit power is $I_{n,m}^{PU}$,

$$I_{n,m}^{PU} = \int_{f_m - f_0 - (n-1)\frac{W}{N}}^{f_m + W_m - f_0 - (n-1)\frac{W}{N}} g_{n,m}^{PU} \phi(f) df, \quad (2)$$

where $g_{n,m}^{PU}$ is the channel gain from the base station of the CR network to the m th PU's receiver. The interference introduced to the n th OFDM subchannel by the m th PU is $I_{n,m}^{SU}$, which is regarded as noise and can be measured by the receivers of the SUs. Denote the signal-to-noise ratio (SNR) of the n th OFDM subchannel with unit power as h_n ,

$$h_n = \frac{g_n^{SU}}{N_0 \frac{W}{N} + \sum_{m=1}^M I_{n,m}^{SU}},$$

where N_0 is the PSD of the additive white Gaussian noise, g_n^{SU} is the power gain between the base station and the SU's receiver. The concerned problem is formulated as

$$\begin{aligned} & \max_{p_n} \sum_{n=1}^N \frac{W}{N} \log_2 \left(1 + \frac{1}{\gamma} p_n h_n \right) \\ \text{s.t. } & C_1 : \sum_{n=1}^N p_n \leq P^T \\ & C_2 : p_n \geq 0, n = 1, 2, \dots, N \\ & C_3 : \sum_{n=1}^N p_n I_{n,m}^{PU} \leq I_m, m = 1, 2, \dots, M, \end{aligned} \quad (3)$$

where γ is the SNR gap, which can be represented as $\gamma = -\frac{\ln(5BER)}{1.5}$ for an uncoded multilevel quadrature amplitude modulation with a specified bit error rate [6]. p_n is the power of the n th subchannel. C_1 and C_2 are the transmission power constraints of the base station of the CR system, where the maximum transmission power is P^T . C_3 contains the interference constraints of the PUs.

III. FAST ALGORITHM

It is easy to prove that (3) defines a convex optimization problem with N variables and $N + M + 1$ constraints [7]. Generally the global optimal solution of the problem can be obtained by standard convex optimization techniques which

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typically have a complexity of $\mathcal{O}(N^3)$. We analyze the structure of the optimization problem and propose a fast barrier method to solve this problem with complexity $\mathcal{O}(M^2N)$.

With the barrier method, the objective problem is converted into a sequence of unconstrained minimization problems by defining a logarithmic barrier function with parameter t , which decides the accuracy of the approximation. These unconstrained optimization problems are solved by Newton method with complexity $\mathcal{O}(N^3)$, where the major cost is spent in the inversion of Hessian during the Newton step.

The barrier function of (3) is

$$\begin{aligned} \phi(P) = & - \sum_{n=1}^N \log(p_n) - \log(P^T - \sum_{n=1}^N p_n) \\ & - \sum_{m=1}^M \log(I_m - \sum_{n=1}^N p_n I_{n,m}^{PU}), \end{aligned} \quad (4)$$

where $P = (p_1, p_2, \dots, p_N)$. Denote

$$f(P) = \sum_{n=1}^N \frac{W}{N} \log_2 \left(1 + \frac{1}{\gamma} p_n h_n \right),$$

the optimal solution of (3) can be approximated by solving the following unconstrained minimization problem [7]

$$\min \psi_t(P) = -tf(P) + \phi(P), \quad (5)$$

where $t > 0$. This is an unconstrained minimization problem that can be solved efficiently by Newton method [7]. As t increases, the approximation becomes more and more accurate.

The computational load of the barrier method mainly lies in the computation of the Newton step ΔP_{nt} at P , that is, solving the equation

$$\nabla^2 \psi_t(P) \Delta P_{nt} = -\nabla \psi_t(P), \quad (6)$$

where $\nabla^2 \psi_t(P)$ is the Hessian and $\nabla \psi_t(P)$ is the gradient of $\psi_t(P)$, respectively. Generally, solving (6) has a cost of $\mathcal{O}(N^3)$. The Hessian of the problem has the following form,

$$\nabla^2 \psi_t(P) = H + \sum_{m=0}^M g_m g_m^T, \quad (7)$$

where

$$H = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_N \end{bmatrix},$$

$\lambda_i = t \frac{W}{N} \frac{h_i^2}{(\gamma + h_i p_i)^2} + \frac{1}{p_i^2}$, and g_i 's are vectors with length N ,

$$g_0 = \left(\frac{1}{P^T - P^N}, \frac{1}{P^T - P^N}, \dots, \frac{1}{P^T - P^N} \right)^T,$$

$$g_m = \frac{1}{I_m - I_m^{tot}(p)} (I_{1,m}^{PU}, I_{2,m}^{PU}, \dots, I_{N,m}^{PU})^T,$$

where $I_m^{tot}(P) = \sum_{n=1}^N p_n I_{n,m}^{PU}$, $P^N = \sum_{n=1}^N p_n$. Define $H_i =$

$H + \sum_{m=0}^i g_m g_m^T$, $i = 0, 1, 2, \dots, M$, we have the following theorem:

TABLE I
TOTAL NUMBER OF ALLOCATED BITS IN THE CR SYSTEM WITH
 $P^T = 1W$, $M = 2$.

N	32	64	128	256	512	1024
This Work	143	252	449	810	1252	1888
Ref. [4]	141	250	444	802	1238	1863
Ref. [5]	139	244	434	783	1211	1829

Theorem 1: All H_i 's are positive definite.

proof: H is diagonal and $\lambda_i > 0$, so H is obviously positive definite. $g_0 g_0^T$ is positive semidefinite, then $H_0 = H + g_0 g_0^T$ is positive definite. Since $g_m g_m^T$ is always positive semidefinite, H_i 's are positive definite sequentially.

It follows that the matrix of (6) is invertible. Since the Hessian H_M can be treated as the sum of a diagonal matrix and $M + 1$ number of rank-one matrices, we can use this special structure to calculate the Newton step ΔP_{nt} with approximate linear complexity.

Theorem 2: The equation (6) can be solved with a complexity of $\mathcal{O}(M^2N)$.

The proof is placed in the Appendix. Recall that M is the number of PUs and generally $M \ll N$ in CR systems, so the complexity of the algorithm is almost linearly related to N .

IV. SIMULATIONS AND DISCUSSIONS

Considering an OFDM-based CR system, each PU occupies random bandwidth which spans continuous subchannels. The noise power is $10^{-13}W$. The interference thresholds of all PUs are set to $5 \times 10^{-13}W$. The channel suffers from frequency selective fading. The path loss exponent is 4. The variance of logarithmic normal shadow fading is 10dB and the amplitude of multipath fading is Rayleigh. The parameters of the barrier method are set to the typical values discussed in [7] and initialized with a strictly feasible solution generated by

$$p_n = \min\{P^T/N, \min_n \{\min_m \{I_m/I_{n,m}^{PU}\}\}\}. \quad (8)$$

First we investigate the convergence performance of the proposed barrier method, of which the criterion is usually the number of Newton iterations [7]. Fig.1 shows the cumulative distribution function of the number of Newton iterations over 1000 instances with $M = 2$, $N = 1024$ and $P^T = 1W$. For 95% instances, Newton iterations for convergence are less than 50 as can be seen from Fig.1. The average number of Newton iterations over 1000 instances is 35.

Table I shows the sum capacity of the CR system as the number of subchannels increases with $P^T = 1W$ and $M = 2$. For comparison with [4] and [5], the sum capacity is measured by the total number of allocated bits of N subchannels. Our proposed algorithm performs the best as seen from Table I. In fact, our method always produces the optimal solutions. It is remarkable that our proposed algorithm has a significant capacity gain if compared with the method in [5].

Fig.2 shows the average time cost as a function of number of subchannels for 1000 instances with $P^T = 1W$ and $M = 2$. Our proposed algorithm has a linear complexity of $\mathcal{O}(N)$. The algorithm in [4] consumes much more time than our method while it only produces suboptimal solutions. Though the time

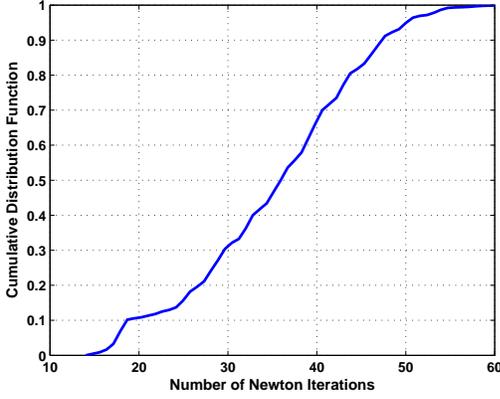


Fig. 1. CDF of number of Newton iterations over 1000 instances with $M = 2$, $N = 1024$ and $P^T = 1W$.

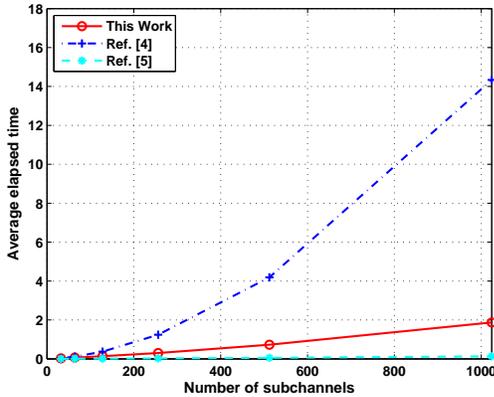


Fig. 2. Average elapsed time as a function of number of subchannels with $P^T = 1W$, $M = 2$.

cost of our method is slightly higher than [5], it is worth noting that our method has achieved a large sum capacity gain. After all, the time cost of the two methods is the same order.

V. CONCLUSIONS

In this paper we studied the optimal power allocation problem in OFDM-based cognitive radio networks and proposed a fast algorithm to achieve the optimal solution. Our proposed algorithm reduces the computational complexity from $O(N^3)$ to $O(M^2N)$, where $M \ll N$ and M is usually small, which makes it promising for practical applications.

APPENDIX

Proof of Theorem 2: Rewrite (6) as

$$H_M u^0 = -\nabla \psi_t(P) \quad (9)$$

where $u^0 = \Delta P_{nt}$. Recall $H_M = H_{M-1} + g_M g_M^T$, (9) can be written as

$$(H_{M-1} + g_M g_M^T) u^0 = -\nabla \psi_t(P) \quad (10)$$

Since H_i 's are positive definite and invertible, then

$$u^0 = (H_{M-1} + g_M g_M^T)^{-1} (-\nabla \psi_t(P)) \quad (11)$$

Using matrix inversion lemma [8], we have

$$u^0 = H_{M-1}^{-1} (-\nabla \psi_t(P)) - \frac{g_M^T H_{M-1}^{-1} (-\nabla \psi_t(P))}{1 + g_M^T H_{M-1}^{-1} g_M} H_{M-1}^{-1} g_M \quad (12)$$

Step 1: Denote two intermediate variables $u_1^1, u_2^1 \in R^n$ as the solutions of the following two sets of linear equations,

$$\begin{aligned} H_{M-1} u_1^1 &= -\nabla \psi_t(P) \\ H_{M-1} u_2^1 &= g_M \end{aligned} \quad (13)$$

then (12) can be written as

$$u^0 = u_1^1 - \frac{g_M^T u_1^1}{1 + g_M^T u_2^1} u_2^1 \quad (14)$$

It means that u^0 can be worked out if u_1^1 and u_2^1 have been calculated.

Step 2: Similarly, u_1^1, u_2^1 can be obtained by solving the following three sets of linear equations,

$$\begin{aligned} H_{M-2} u_1^2 &= -\nabla \psi_t(P) \\ H_{M-2} u_2^2 &= g_M \\ H_{M-2} u_3^2 &= g_{M-1} \end{aligned} \quad (15)$$

where $u_1^2, u_2^2, u_3^2 \in R^n$ are other intermediate variables.

Continue this process to *Step M + 1*, $M + 1$ variables $u_1^M, u_2^M, \dots, u_{M+1}^M \in R^n$ are obtained by solving $M + 2$ sets of linear equations,

$$\begin{aligned} H u_1^{M+1} &= -\nabla \psi_t(P) \\ H u_2^{M+1} &= g_M \\ &\vdots \\ H u_{M+2}^{M+1} &= g_0 \end{aligned} \quad (16)$$

Since H is diagonal, each set of equations in (16) can be solved at a cost of $O(N)$. The computation cost of solving (16) is $O(MN)$. Using (14), we calculate all $u_i^M, i = 1, 2, \dots, M + 1$ with $O(MN)$ complexity. Carry out the iteration process inversely, we can calculate all the intermediate variable $u_1^{i-1}, u_2^{i-1}, \dots, u_i^{i-1}$ with a cost of at most $O(MN)$ until u^0 is worked out. The total cost is $O(M^2N)$. Notice that all H_i 's are positive definite, the condition of using the matrix inversion lemma is always satisfied during the computations.

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