DFS Search on Undirected Graphs

Algorithm : Design & Analysis

[13]

In the last class…

- Directed Acyclic Graph
  - Topological Order
  - Critical Path Analysis
- Strongly Connected Component
  - Strong Component and Condensation
  - Leader of Strong Component
  - The Algorihtm

DFS Search on Undirected Graph

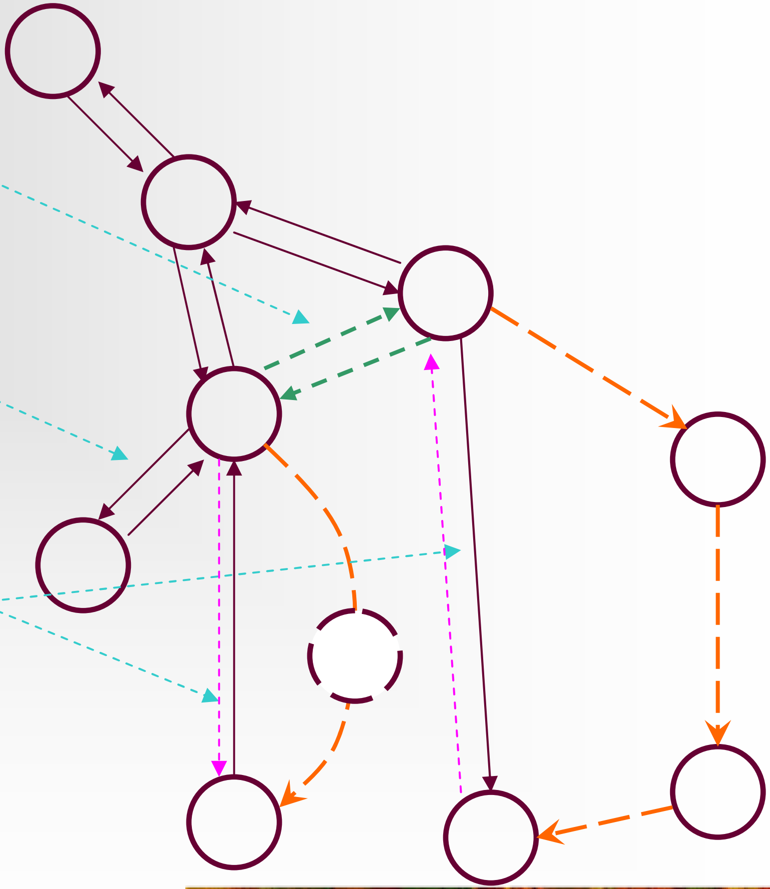
- Undirected and Symmetric Digraph
- UDF Search Skeleton
- Biconnected Components
  - Articulation Points and Biconnectedness
  - Biconnected Component Algorithm
  - Analysis of the Algorithm

What’s the Different for Undirected

- The issue related to traversals for undirected graph is that **one edge may be traversed for two times in opposite directions**.
- For an undirected graph, the depth-first search provides an orientation for each of its edges; they are oriented in the direction in which they are first encountered.

Nontree edges in symmetric digraph

- Cross edge: not existing.
- Back edge:
  - Back to the direct parent: second encounter
  - Otherwise: **first encounter**
- Forward edge: always second encounter, ***and first time as back edge***



Modifications to the DFS Skeleton

- All the second encounter are bypassed.
- So, the ***only substantial modification*** is for the possible back edges leading to an ancestor, but not direct parent.
- We need know the ***parent***, that is, the direct ancestor, for the vertex to be processed.

DFS Skeleton for Undirected Graph

- **int** dfsSweep(IntList[] *adjVertices*,**int** n, …)
- **int** ans;
- <Allocate color array and initialize to white>
- For each vertex *v* of G, in some order
- **if** (color[v]==white)
- **int** vAns=**dfs(*adjVertices*, color, v, -1, …);**
- <Process vAns>
- // Continue loop
- **return** ans;

Recording the parent

DFS Skeleton for Undirected Graph

```
int dfs(IntList[] adjVertices, int[] color, int v, int p, …)
    int w; IntList remAdj; int ans;
    color[v]=gray;
    <Preorder processing of vertex v>
    remAdj=adjVertices[v];
    while (remAdj≠nil)
        w=first(remAdj);
        if (color[w]==white)
            <Exploratory processing for tree edge vw>
            int wAns=dfs(adjVertices, color, w, v …);
            < Backtrack processing for tree edge vw , using wAns>
        else if (color[w]==gray && w≠p)
            <Checking for nontree edge vw>
        remAdj=rest(remAdj);
    <Postorder processing of vertex v, including final computation of ans>
    color[v]=black;
    return ans;
```

In all other cases, the edges are the second encounter, so, ignored.

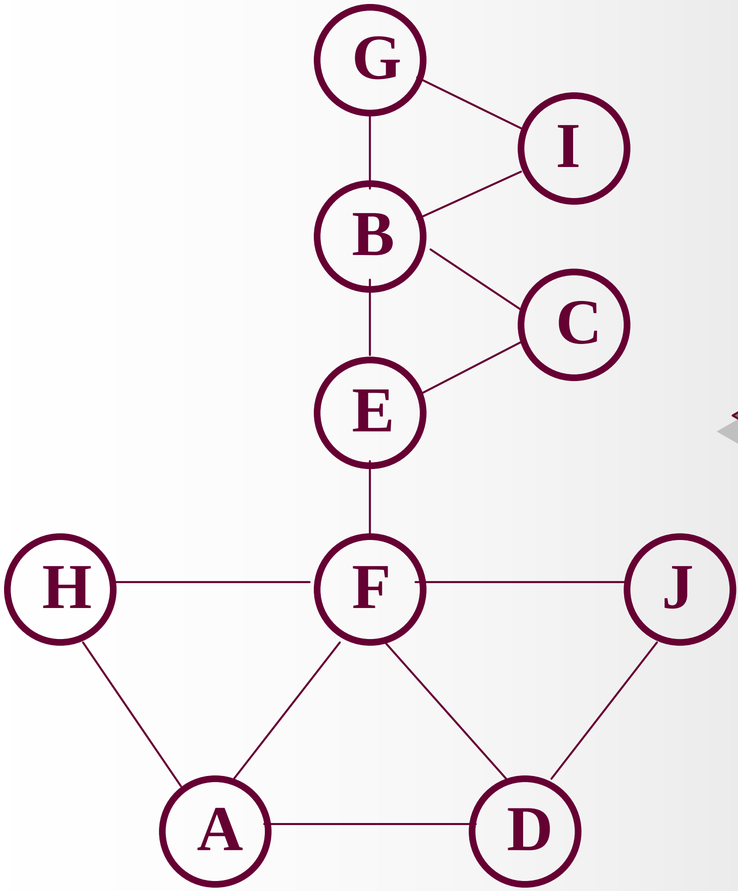
Complexity of Undirected DFS

- If each inserted statement for specialized application runs in constant time, the time cost is the same as for directed DFS, that is $\Theta(m+n)$.
- Extra space is in $\Theta(n)$ for array *color*, or activation frames of recursion.

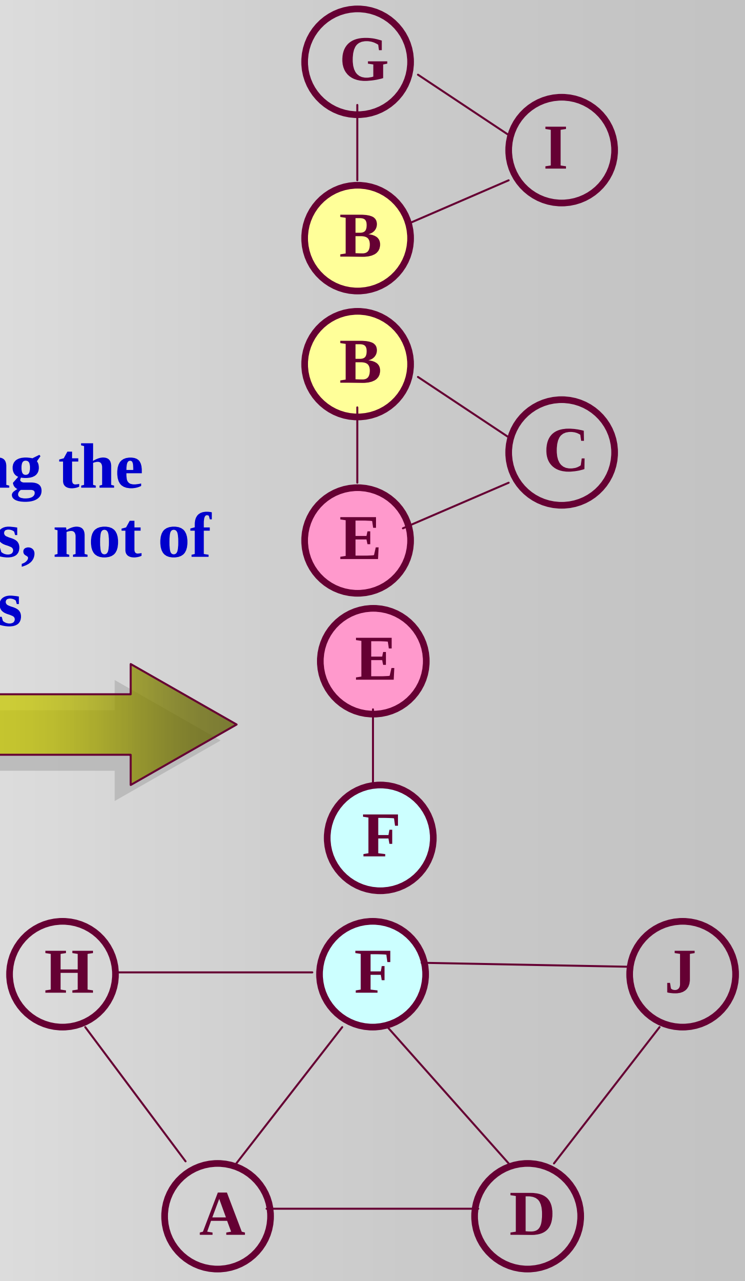
Definition of Biconnected Components

- Biconnected component
  - Biconnected graph
  - Bicomponent: a maximal biconnected subgraph
- Articulation point
  - *v* is an articulation point if it is in **every** path from *w* to *x* (*w*,*x* are vertices different from *v*)
- A connected graph is biconnected if and only if it has no articulation points.

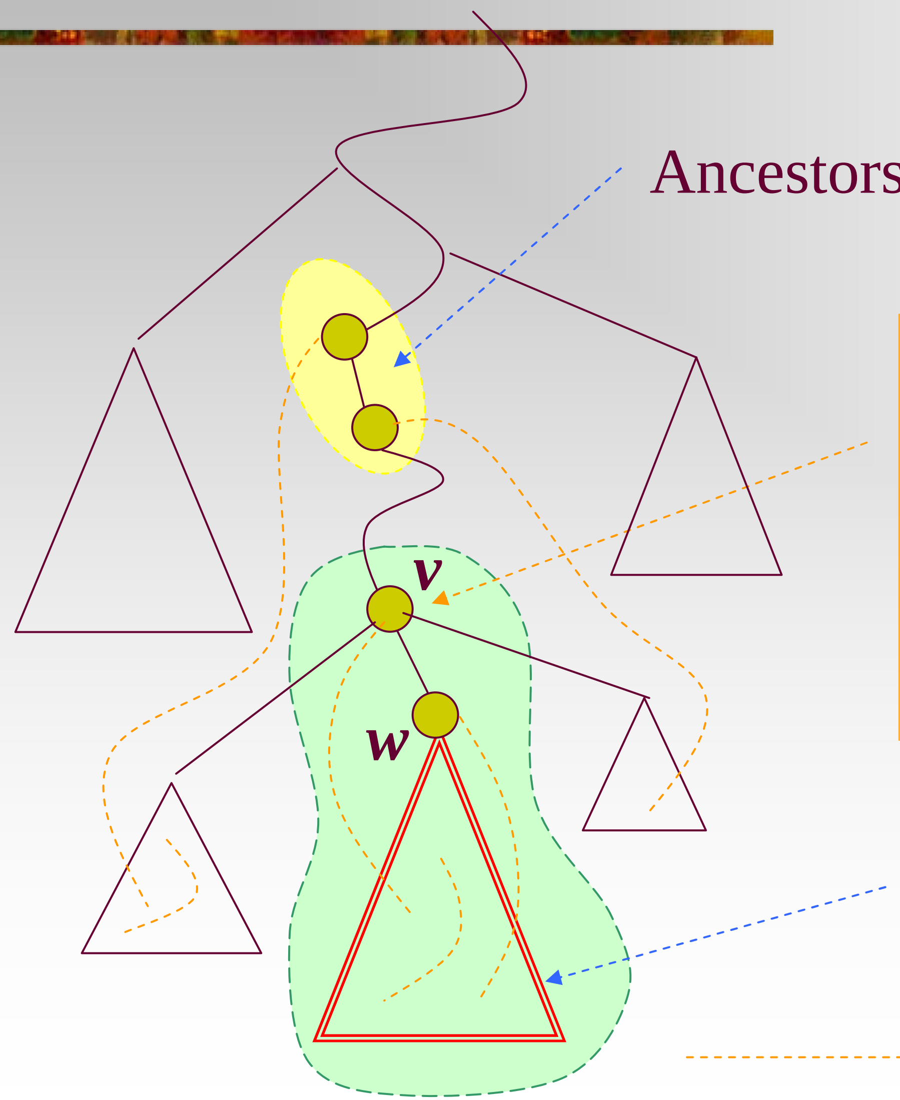
Bicomponents



Partitioning the set of edges, not of the vertices



Bicomponent Algorithm: the Idea



v is an articulation point

If *v* is the articulation point farthest away from the root on *the* branch, then one bicomponent is detected.

Keeping the Track of Backing

- Tracking data
  - For each vertex *v*, a local variable *back* is used to store the required information, as the value of *discoverTime* of some vertex.
- Testing for bicomponent
  - At backtracking from *w* to *v*, the condition implying a bicomponent is:

$$wBack \geq discoverTime(v)$$

(where *wBack* is the returned back value for *w*)

Updating the va

- *v* first discovered

  ***back*=**

- Trying to explore, b

  encountered

  ***back*=min(*back*, *discoverTime*(*w*))**

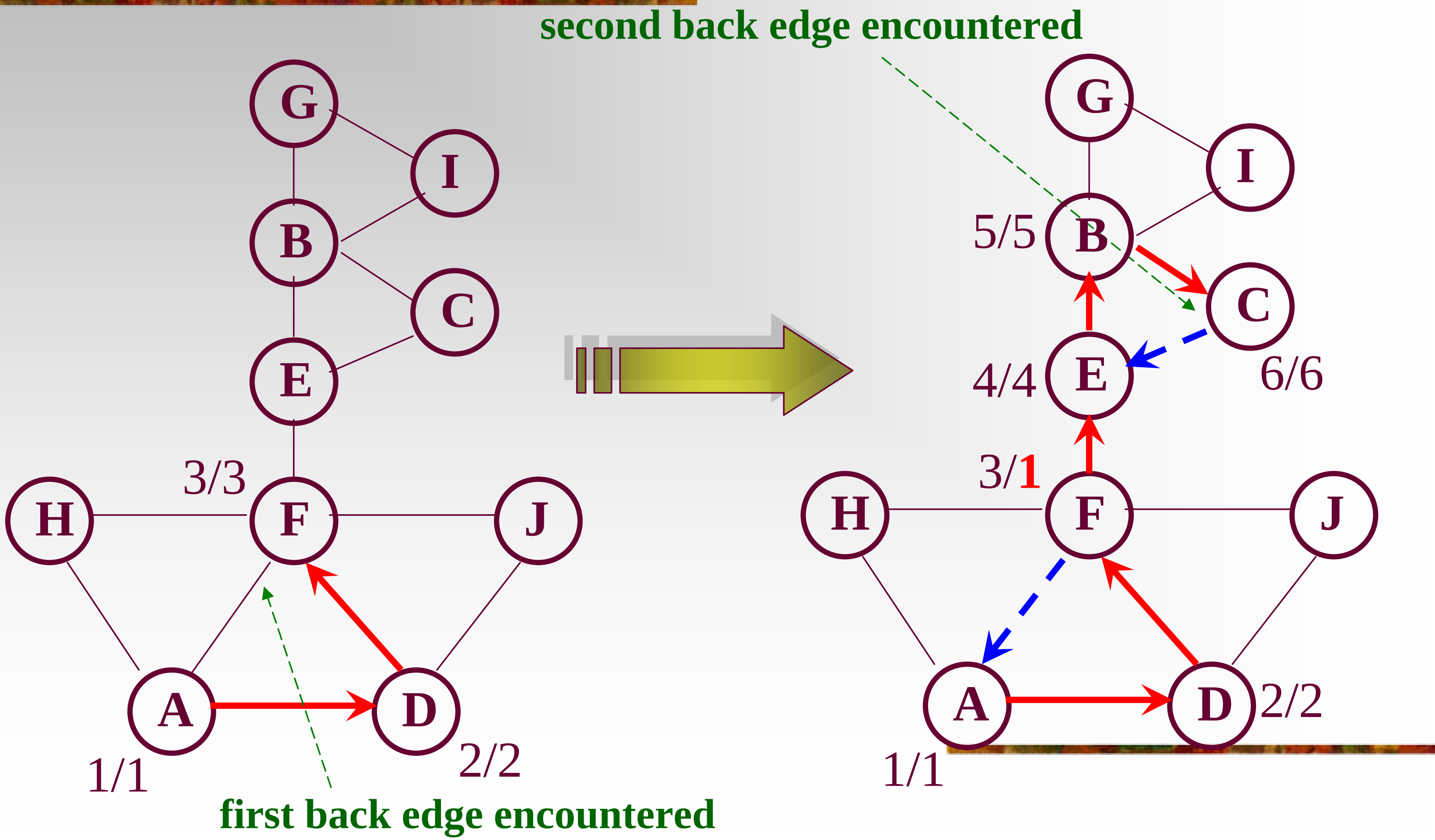
- *Backtracking from w to v*

  ***back*=min(*back*, *wback*)**

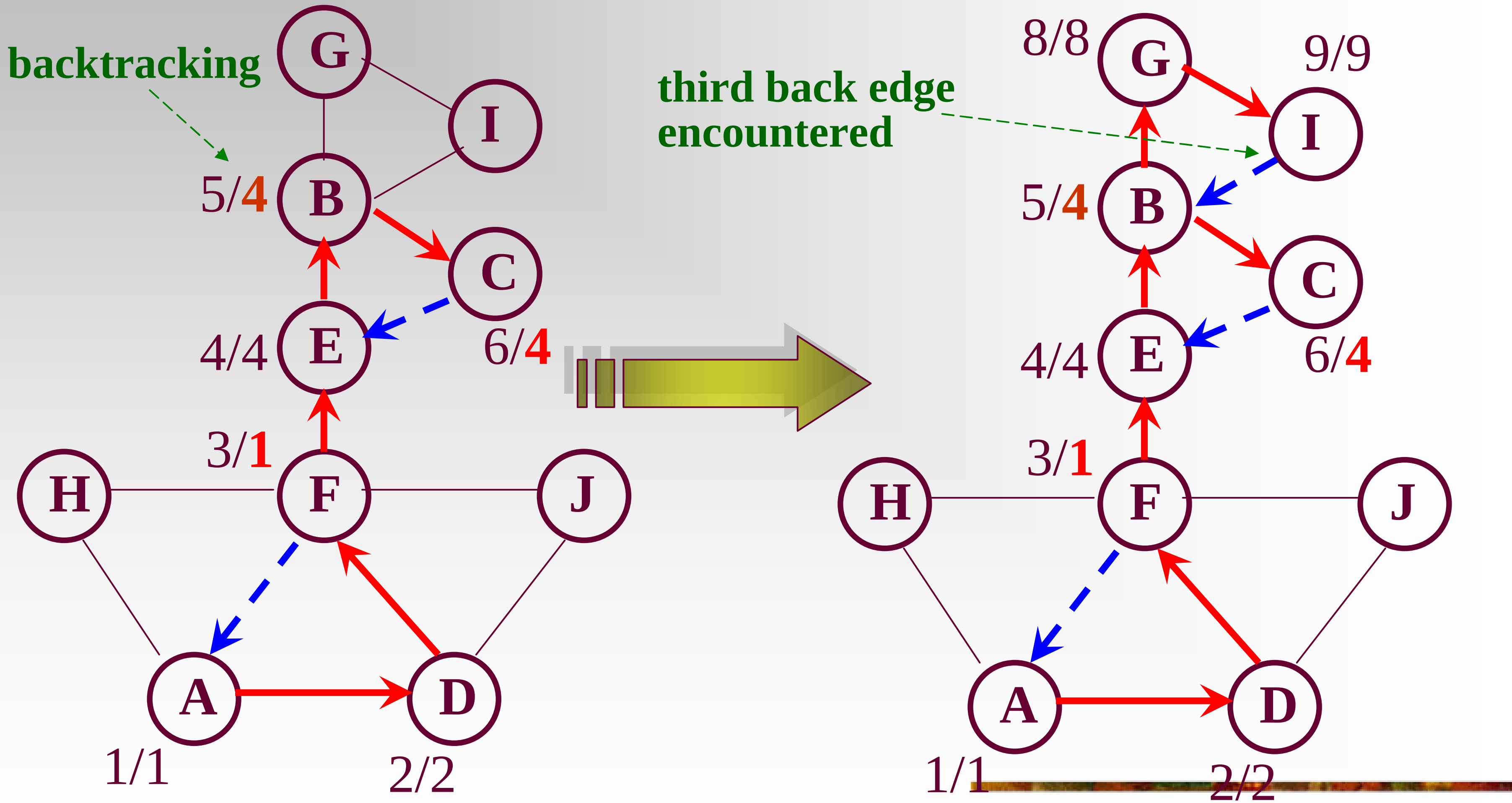
Which means: the back value of *v* is the smallest discover time a back edge "sees" from **any** subtree of *v*.

And, when this value is not larger than the discover time of *v*, we know that there is at least one subtree of *v* connected to other part of the graph only by *v*.

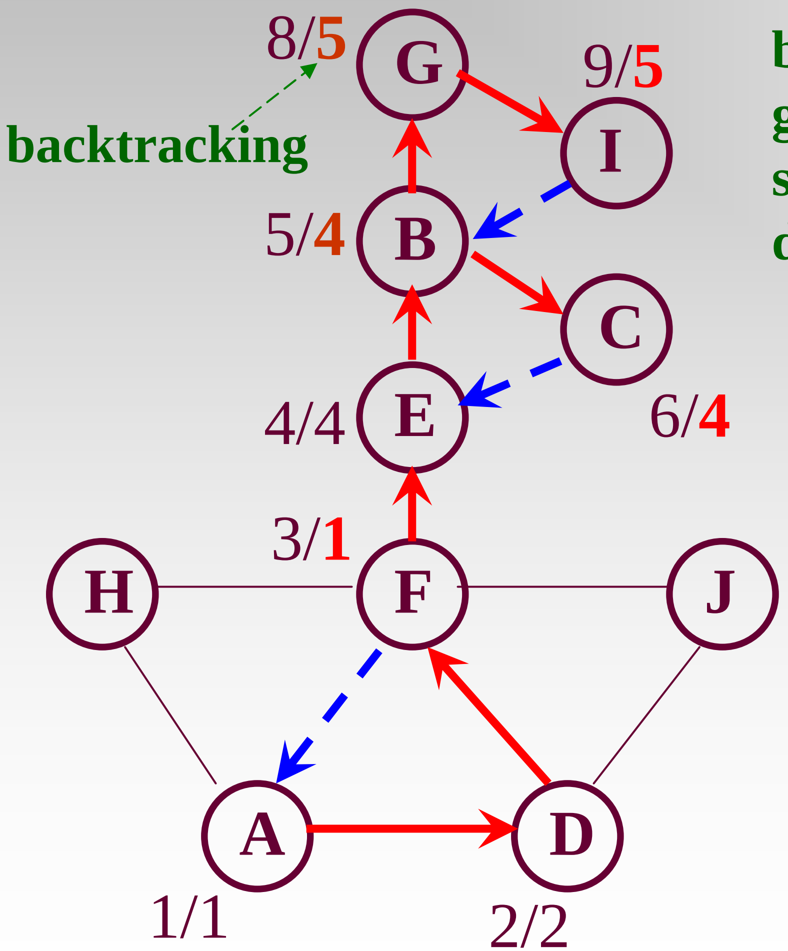
Bicomponent: an Example



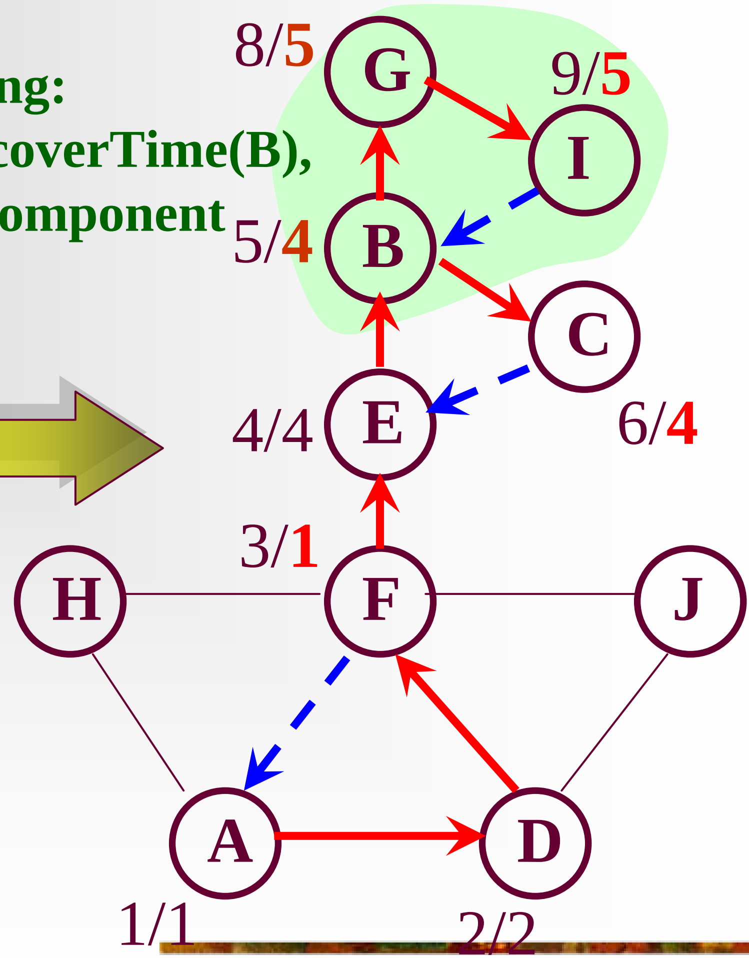
Bicomponent: an Example



Bicomponent: an Example

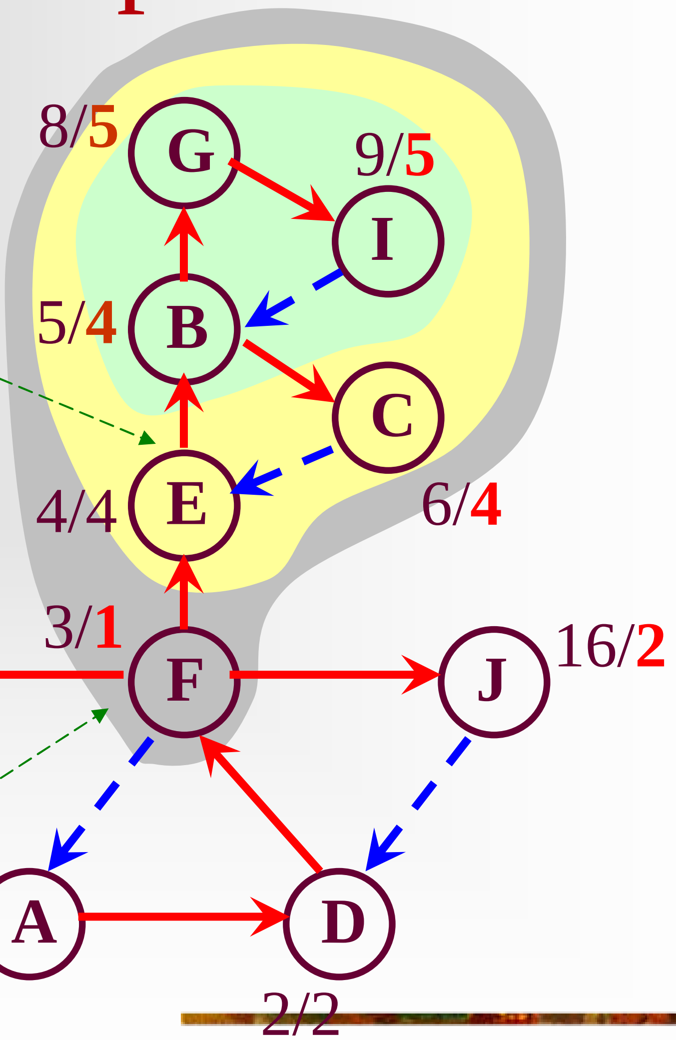


backtracking:
gBack=discoverTime(B),
so, first bicomponent
detected.



Bicomponent: an Example

Backtracking from B to E:
bBack=discoverTime(E), so,
the second bicomponent is
detect



Backtracking from E to F:
eBack>discoverTime(F), so,
the third bicomponent is detect

Bicomponent Algorithm: Core

```
int bicompDFS(v)
    color[v]=gray; time++; discoverTime[v]=time;
    back=discoverTime[v];
    while (there is an untraversed edge vw)
        <push vw into edgeStack>
        if (vw is a tree edge)
            wBack=bicompDFS(w);
            if (wBack≥discoverTime[v])
                Output a new bicomponent
                    by popping edgeStack down through vw ;
            back=min(back, wBack);
        else if (vw is a back edge)
            back=min(discoverTime[v], back);
    time++; finishTime[v]=time; color[v]=black;
return back;
```

Outline of core procedure

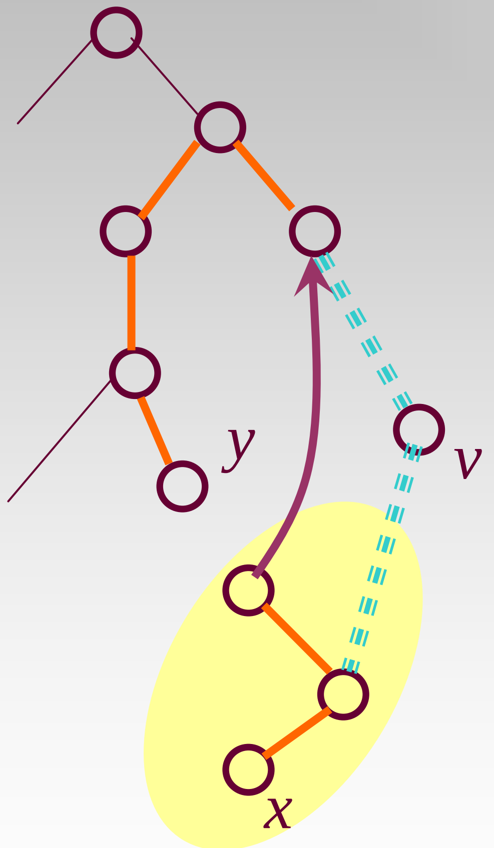
Correctness of Bicomponent Algorithm

- We have seen that:
  - If v is the articulation point farthest away from the root on the branch, then one bicomponent is detected.
- So, we need only prove that:
  - In a DFS tree, a vertex(not root) *v* is an articulation point **if and only if** (1)*v* is not a leaf; (2) **some** subtree of *v* has **no back edge** incident with a proper ancestor of *v*.

Characteristics of Articulation Point

- In a DFS tree, a vertex(not root) v is an articulation point **if and only if** (1)v is not a leaf; (2) **some** subtree of v has **no back edge** incident with a proper ancestor of v.
- $\Leftarrow$ Trivial
- $\Rightarrow$
  - By definition, v is on **every** path between some x,y(different from v).
  - At least one of x,y is a proper descendent of v(otherwise, $x \leftrightarrow root \leftrightarrow y$ not containing v).
  - By contradiction, suppose that **every** subtree of v has a back edge to a proper ancestor of v, we can find a xy-path not containing v for all possible cases(only 2 cases)

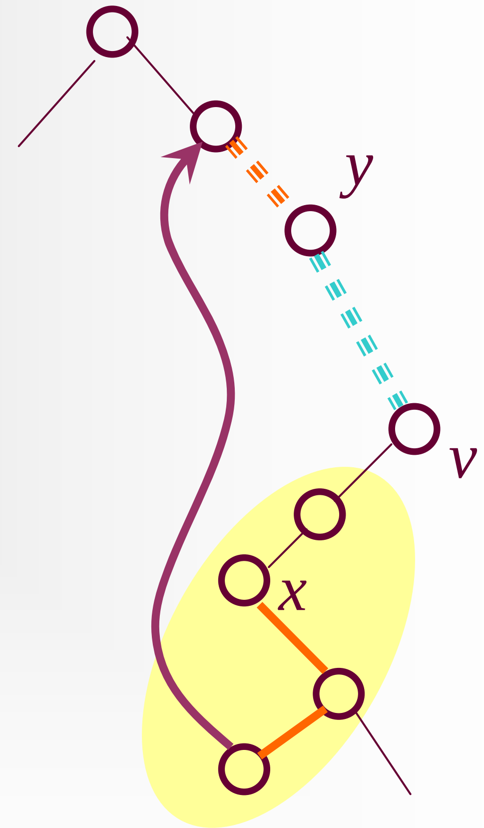
Case 1



Case 1.1: another is not an ancestor of *v*

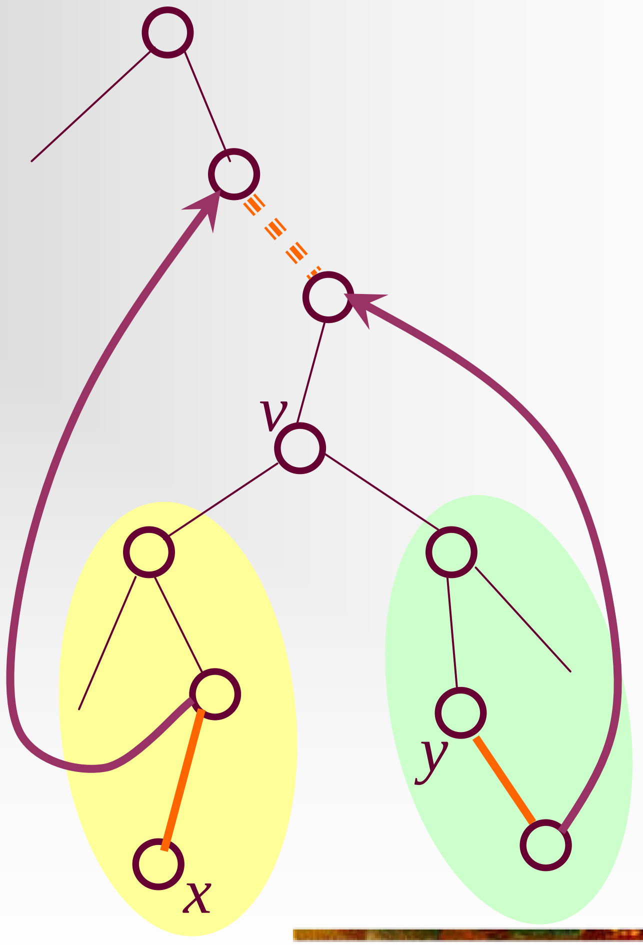
suppose that **every** subtree of *v* has a back edge to a proper ancestor of *v*, and, exactly one of *x*, *y* is a descendant of *v*.

Case 1.2: another is an ancestor of *v*



Case 2

suppose that **every** subtree of *v* has a back edge to a proper ancestor of *v*, and, both *x*, *y* are descendants of *v*.



Home Assignments

- pp.380-
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  - 7.38
  - 7.40