Information Coverage for Wireless Sensor Networks

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***Abstract*— Coverage is a very important issue in wireless sensor networks. Current literature defines a point to be covered if it is within the sensing radius of at least one sensor. In this paper we argue that this is a conservative definition of coverage. This definition implicitly assumes that each sensor makes a decision independent of other sensors in the field. However, sensors can cooperate to make an accurate estimation, even if any single sensor is unable to do so. We then propose a new notion of *information coverage* and investigate its implications for sensor deployment. Numerical and simulation results show that significant savings in terms of sensor density for complete coverage can be achieved by using our definition of information coverage compared to that by using the existing definition.**

***Index Terms*— Coverage, information coverage, sensor networks, estimation.**

I. Introduction

SENSOR coverage is an important issue in wireless sensor networks [1][2]. In most sensor network literature, every sensor node is assumed to have a fixed sensing accuracy and sensing radius [3][4][5]. A point is said to be covered if its Euclidean distance to a sensor is within the sensing radius of the sensor. This notion of coverage is referred to as *physical coverage* and the point is said to be *physically covered* in this paper. Under the notion of physical coverage, all points in a field should be within the sensing distance of at least one sensor to achieve complete coverage for the field. However, this might not be necessary under other notions of coverage. Consider an event driven application, e.g. target detection. We note here that some parameters of the event (e.g., acoustic readings from tank movement) decay with the distance to the position at which the event occurs. In this case, even the sensor closest to the position of the event might not be able to make an accurate estimate of the event by itself. However, several sensors can cooperate to make an accurate estimate of the event.

These observations motivate us to reexamine our notion of coverage. We explore the notion of *information coverage* via estimation theory, where sensors cooperate to make an estimate or decision for the data to sense at a particular location. We also analyze some properties of the proposed information coverage concept and present numerical examples to illustrate that the sensor density can be greatly reduced with information coverage compared to physical coverage.

II. Estimation in Sensor Networks

Consider a set of K geographically distributed sensors, each making a measurement on an unknown parameter θ at some location and time. We assume that each sensor knows its own coordinates. An example can be the sensing of an acoustic signal of amplitude θ. Let $d_k, k = 1, 2, ..., K$ denote the distance between a sensor k and a location with some parameter θ. The parameter θ is assumed to decay with distance, and at distance d it is θ/d^{α}, where $\alpha > 0$ is the decay exponent. The above sensing model has been used in [6] to determine path exposure. The measurement of the parameter, x_k, at a sensor may also be corrupted by an additive noise, n_k. Thus

$$x_k = \frac{\theta}{d_k^{\alpha}} + n_k, k = 1, 2, ..., K. \tag{1}$$

The objective of a parameter estimator is to estimate θ based on the corrupted measurements. Let $\hat{\theta}$ and $\tilde{\theta} = \hat{\theta} - \theta$ denote the estimate and the estimation error, respectively. When K measurements are available, a well-known *best linear unbiased estimator* (BLUE) [7] can be applied to estimate $\hat{\theta}_K$ and to achieve a minimum *mean squared error* (MSE).

The measurement given by (1) can be written in matrix format for K sensors as $\mathbf{X} = \mathbf{D}\theta + \mathbf{N}$, where $\mathbf{X} = (x_1, x_2, ..., x_K)^T$, $\mathbf{D} = (d_1^{-\alpha}, d_2^{-\alpha}, ..., d_K^{-\alpha})^T$, and $\mathbf{N} = (n_1, n_2, ..., n_K)^T$. The additive noises are assumed to be spatially uncorrelated white noise with zero mean and σ_k^2 variance, but otherwise unknown. The covariance matrix of the noises $\{n_k : 1, 2, ..., K\}$ is given by $\mathbf{R} = \mathbb{E}[\mathbf{N}\mathbf{N}^T] = \mathrm{diag}[\sigma_1^2, \sigma_2^2, ..., \sigma_K^2]$. According to BLUE, when K measurements are available, the estimate $\hat{\theta}_K$ of the original signal θ is given by

$$\hat{\theta}_K = \left[\mathbf{D}^T\mathbf{R}^{-1}\mathbf{D}\right]^{-1}\mathbf{D}^T\mathbf{R}^{-1}\mathbf{X}. \tag{2}$$

and the estimation error $\tilde{\theta}$ is given by

$$\tilde{\theta}_K = \hat{\theta}_K - \theta = [\mathbf{D}^T\mathbf{R}^{-1}\mathbf{D}]^{-1}\mathbf{D}^T\mathbf{R}^{-1}\mathbf{N}. \tag{3}$$

III. Information Coverage Based on Estimation

If an event that has occurred at a particular location can be estimated with a guaranteed estimation error, this location can be considered as being monitored by these cooperative sensors. Note that the estimation error $\tilde{\theta}_K$ given by (3) is a random variable with zero mean (due to the zero mean uncorrelated noises) and variance $\tilde{\sigma}_K^2$. The probability that the absolute value of the estimation error being less than a constant A is larger than a given threshold ϵ $(0 < \epsilon < 1)$ is given by

$$\Pr[|\tilde{\theta}_K| \le A] \ge \epsilon. \tag{4}$$

Based on (4), we define information coverage for K cooperative sensors as follows.

Definition 1: (K**-sensor** ϵ**-error information coverage**) *A point is said to be* (K, ϵ)*-covered if any parameter on this point can be estimated by* K *sensors such that the probability*

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that the absolute value of the estimation error is equal to or less than a constant A is equal to or larger than ϵ, $0 \leq \epsilon \leq 1$. A region is said to be completely (K, ϵ)-covered if all the points of the region are (K, ϵ)-covered.

A sensor is called *isotropic* if its sensing ability is the same in all directions. For such a sensor, its $(1, \epsilon)$ information coverage is a disk centered at the sensor. We can set the disk radius as the maximum distance between a sensor and a point such that the point can be $(1, \epsilon)$-covered. In this case, the $(1, \epsilon)$ information coverage is the same as the physical coverage.

When examining (K, ϵ) information coverage for a point, one can first check $(1, \epsilon)$ coverage for this point. If it is $(1, \epsilon)$-covered, then it is also (K, ϵ)-covered. If it is not $(1, \epsilon)$-covered, one more sensor is added to examine if it is $(2, \epsilon)$ covered and so on. This is because if a point is (k, ϵ)-covered, it is also $(k + 1, \epsilon)$-covered. Another interesting issue is how to efficiently choose K sensors to provide the information coverage. Since BLUE is an unbiased estimator (i.e., zero mean estimation error) to minimize the mean squared error, we can use the variance of the estimation error to measure the estimation efficiency. Let $\overline{K} = (k_1, k_2, ..., k_K)$ denote a sequence of K sensors. The following definition gives an efficiency measurement for sensor selection, and Theorem 1 provides a necessary condition for the most efficient sequence.

Definition 2: A sequence of sensors $\overline{K} = (k_1, k_2, ..., k_K)$ is said to be more efficient than another sequence $\overline{K}' = (k'_1, k'_2, ..., k'_K)$ if $\mathbb{V}[\tilde{\theta}_{k_i}] \leq \mathbb{V}[\tilde{\theta}_{k'_i}]$, for all $i = 1, 2, ..., K$, where $\mathbb{V}$ denotes the variance.

Theorem 1: The most efficient sequence of sensors $\overline{K} = (k_1, k_2, ..., k_K)$ should satisfy

$$d_{k_1}^{2\alpha}\sigma_{k_1}^2 \leq d_{k_2}^{2\alpha}\sigma_{k_2}^2 \leq \cdots \leq d_{k_K}^{2\alpha}\sigma_{k_K}^2. \quad (5)$$

Proof: The proof proceeds by induction on the construction of $\overline{K}$. For $i = 1$, we have $\tilde{\theta}_{k_1} = d_1^{\alpha} n_1$ and hence $\mathbb{V}[\tilde{\theta}_{k_1}] = d_{k_1}^{2\alpha}\sigma_{k_1}^2$. Obviously the smallest $\mathbb{V}[\tilde{\theta}_{k_1}]$ is achieved when choosing the sensor k_1 such that $d_{k_1}^{2\alpha}\sigma_{k_1}^2$ is the smallest among all sensors. Now assume the construction of $\overline{K}$ is the most efficient for i sensors, i.e., the sequence of selected sensors is $(k_1, k_2, ..., k_i)$ such that $d_{k_1}^{2\alpha}\sigma_{k_1}^2 \leq d_{k_2}^{2\alpha}\sigma_{k_2}^2 \leq ... \leq d_{k_i}^{2\alpha}\sigma_{k_i}^2$ and $\mathbb{V}[\tilde{\theta}_{k_i}]$ is the most efficient for the sensor sequence $1, ..., i$. From (3), we have

$$\mathbb{V}[\tilde{\theta}_{k_{i+1}}] = \frac{1}{\frac{1}{\mathbb{V}[\tilde{\theta}_{k_i}]} + \frac{1}{d_{k_{i+1}}^{2\alpha}\sigma_{k_{i+1}}^2}} \quad (6)$$

Consider another sequence construction $\overline{K}'$ which is also the most efficient sequence for the first i sensors. However, the $(i + 1)$th sensor is different from $\overline{K}$. From (6), it is easy to see that $\mathbb{V}[\tilde{\theta}_{k_{i+1}}] \leq V[\tilde{\theta}_{k'_{i+1}}]$ implies that $d_{k_{i+1}}^{2\alpha}\sigma_{k_{i+1}}^2 \leq d_{k'_{i+1}}^{2\alpha}\sigma_{k'_{i+1}}^2$. That is, the selection of the k_{i+1} sensor should also satisfy (5). Hence the desired result is obtained from induction. ■

Corollary 1: When all sensors have the same noise variance, i.e., $\sigma_1^2 = \sigma_2^2 = ... = \sigma_K^2$, the most efficient sequence of sensors $\overline{K} = (k_1, k_2, ..., k_K)$ satisfies $d_{k_1} \leq d_{k_2} \leq \cdots \leq d_{k_K}$.

The proposed concept of information coverage is based on parameter estimation. However, its definition is general and not dependent on any particular estimator. Some other estimators such as weighted least square estimator can also be used for estimation. BLUE is the most efficient estimator within the class of unbiased estimators that are linearly related to the measurements [7]. In some applications or for fault tolerance, a point should be physically covered by more than one sensor (M physical coverage), i.e., its Euclidean distances to M nearest sensors are all within the sensing radius. M physical coverage uses redundant sensors to increase coverage robustness. When up to $M - 1$ sensors fail, the point can still be physically covered. Robustness in information coverage can also be achieved by using redundant sensors. To achieve M robustness, a point should be M $(K, \epsilon)-$covered. This requires that any K out of $K + M$ nearest sensors of a point can (K, ϵ)-cover the point. Another way to achieve robustness is to include more sensors to recover the information coverage for a point. For example, if a point cannot be (K, ϵ)-covered due to sensor failure, we may find another two sensors to $(K + 1, \epsilon)$-cover the point.

IV. Examples of Information Coverage

One of the benefits of using information coverage is to reduce sensor density to completely cover an area. Consider a special case that all noises are Gaussian and independent. The sum of these noises is still Gaussian with zero mean and variance $\tilde{\sigma}_K^2 = \sum_{k=1}^{K} a_k^2\sigma_k^2$, where $a_k = B_K/d_k^{\alpha}\sigma_k^2$ and $B_K = \left(\sum_{k=1}^{K} 1/d_k^{2\alpha}\sigma_k^2\right)^{-1}$. We further assume that all noises have the same variance, i.e., $\sigma_k^2 = \sigma^2$ for all $k = 1, 2...$, and define the sensing range of a single sensor, D_s , as the distance where the estimation error performance equals ϵ. After some algebraic manipulations, D_s satisfies $Q(\frac{A}{D_s^{\alpha}\sigma}) = \frac{1-\epsilon}{2}$, where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp(-\frac{t^2}{2})dt$. Therefore, A can be set as $\beta\sigma$, $\beta > 0$. Here, we set $A = \sigma$ and choose a certain ϵ, and compute D_s as the sensing range used to relate the physical coverage and information coverage in a single sensor case. Another way is to set $A = \sigma$ and set D_s as the unit for distance, and compute ϵ accordingly. Here, we set $D_s = 1$, and hence $\epsilon = 0.683$.

We first use simple regular placements of sensors to compare the physical and information coverage and to investigate the density requirement for complete coverage. Fig. 1 illustrates the physical and information coverage when the tilings are either equilateral triangles or squares. In particular, with regular tilings and $\alpha = 1$, the area of an information covered triangle/square/hexagon is 3.0/4.0/6.0 times larger than that of a physically covered triangle/square/hexagon, respectively. The coverage density is defined as the number of sensors per unit area in order to fully cover the 2-D plane. The node density requirements for information and physical coverage are compared in Table I. It is observed that improvements in sensor density decrease as the decay exponent α increases. This is because the received signal energy decreases dramatically with a larger decay exponent. The density gain when using regular polygons with M ($M = 3, 4, 6$) sides to tile up the field can be approximated by $M^{\frac{1}{\alpha}}$. Thus for typical values $2 \leq \alpha \leq 4$, the range of density gains is $1.32 \leq \rho_1/\rho_2 \leq 1.73$ for triangular tiling, $1.41 \leq \rho_1/\rho_2 \leq 2.0$ for square tiling, and $1.56 \leq \rho_1/\rho_2 \leq 2.45$ for hexagonal tiling.

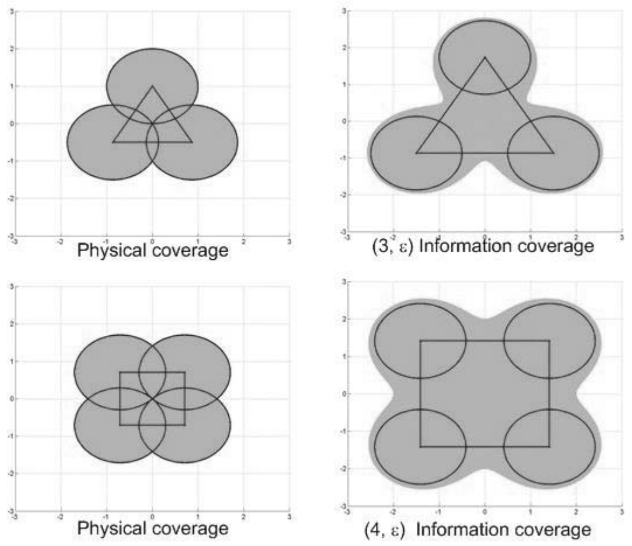

Fig. 1. Comparison of physical and information coverage for regular sensor deployment ($\alpha = 1.0$, $\epsilon = 0.683$).

TABLE I

COMPARISON OF NODE DENSITY FOR REGULAR NODE PLACEMENT

	$\alpha = 1.0$			$\alpha = 2.0$		
Grid types	3	4	6	3	4	6
Physical Density (ρ_1)	0.39	0.5	0.77	0.39	0.5	0.77
Information Density (ρ_2)	0.13	0.13	0.13	0.23	0.25	0.32
ρ_1/ρ_2	3.0	4.0	6.0	1.73	2.0	2.45

Next, we use simulations to compare density requirements for random uniform sensor deployment. To reduce boundary effect, two co-center squares with side length 10 and 14 are used. A grid with 1000×1000 vertices is created for the inner square. The inner square is completely covered if all vertices of the grid are covered. Sensors are randomly scattered within the outer square and this process is repeated 100 times for each number of scattered sensors. The probability of complete (K, ϵ) coverage, $\Pr\{\text{complete } (K, \epsilon) \text{ coverage}\}$, is then defined as the ratio between the number of times all vertices are (K, ϵ)-covered and the number of simulation times (100). Fig. 2 plots $\Pr\{\text{complete } (K, \epsilon) \text{ coverage}\}$ for different numbers of deployed sensors. It is observed that the number of sensors for complete $(2/4/6, \epsilon)$ coverage is only about $\frac{1}{2}/\frac{1}{4}/\frac{1}{6}$ times of that for complete $(1, \epsilon)$ coverage. That is, density requirements for complete information coverage are significantly reduced even for random sensor deployment. The gap between the density for $\Pr\{\text{complete } (K, \epsilon) \text{ coverage}\} = 0$ and the density for $\Pr\{\text{complete } (K, \epsilon) \text{ coverage}\} = 1$ is often referred to as the *window of phase transition* which is frequently used as an indicator for the coverage convergence rate. It is also observed from Fig. 2 that the window of phase transition for information coverage is smaller than that for physical coverage; and the higher the value of K, the smaller the window.

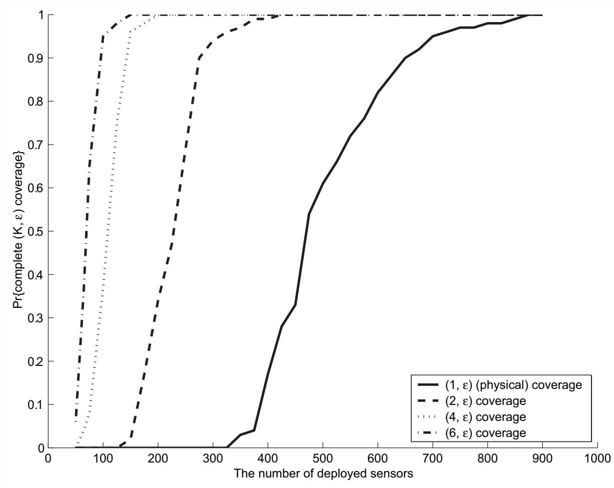

Fig. 2. $\Pr\{\text{complete } (K, \epsilon) \text{ coverage}\}$ vs the number of deployed sensors for random uniform deployment ($\alpha = 1.0$, $\epsilon = 0.683$).

V. CONCLUDING REMARKS

In this paper, we have argued that the notion of physical coverage used in current wireless sensor networks literature is inadequate. The notion of information coverage based on parameter estimation has been proposed and its properties have been analyzed. We have illustrated how substantial reductions in density requirements for complete coverage can be achieved if we use the notion of information coverage. Besides coverage, network cost and other network performances can also be impacted by using the notion of information coverage. For example, with reduced sensor density, network cost can be reduced. Although using information coverage may increase computation cost (battery power), it can still benefit from less communication cost due to reduced density. The impacts on the network performance will be considered in future work.

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