Coverage for Target Localization in Wireless Sensor Networks

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Overview

- Introduction

- Sufficient condition for full coverage for localization applications

- Estimating the necessary network density for localization
  -- Disk coverage model
  -- Sector coverage model

- Conclusion
Coverage problems in sensor networks

- Coverage problem: Find a way to deploy or schedule sensors so that the *information loss* of the sensor network is below a given bound

- Information loss: the differences between the sensed data and the physical world
  -- Depends on applications, can be measured as *target detection rate*, *localization error* or *data sampling rate* (in time or space)

- Conventional coverage models
  -- Disk coverage model
  -- Exposure model
Insufficiency of the disk coverage model

- **Disk model**
  - Sensing area is a disk around sensors
  - A point is deemed to be covered when it is within the sensing area

- **Insufficient for localization**
  - Detected target can be localized with certain accuracy
  - A $k$-covered target still may not be accurately localized
Problems addressed in this paper

- How to guarantee a localization error bound in a sensor network?

- The relationship between sensor density and the localization error in a random network

- Methods for estimating sensor density when given the localization error requirement. Can we apply disk model in this case?
Localization model used in this paper

- The locations for sensors are known
- Sensors can measure its distance to the target
- When the estimated distance is $d_{i,t}^0$, then the true distance between sensor $i$ and target $t$ is within $[d_{i,t}^0 - e, d_{i,t}^0 + e]$ with high probability, where $e$ is the error bound when target is within detection range.
Localizing through multiple sensors

- Each sensor can localize the target within a ring of 2e
- Use the center of the intersection area as the estimated target location
- Localization error depends on the size of intersection area
Network resolution

- The network can distinguish any pair of two points, which are departed more than the network resolution, through the distance measurements.

- Relationship between network resolution and localization error:
  - $\text{Network resolution} < \sqrt{3} \epsilon \quad \Rightarrow \quad \text{Localization error} < \epsilon$
  - $\text{Network resolution} > 2 \epsilon \quad \Rightarrow \quad \text{Localization error} > \epsilon$
Sufficient condition for localization coverage

**Theorem 1:**

If there is no \((r, \frac{2}{3})\) sector shaped void in the network

Every pair of points can be distinguished by at least one sensor when they are depart by more than 4\(e\). Thus the network resolution is smaller than 4\(e\)

The localization error over the network is bounded by \(\frac{4\sqrt{3}}{3} e\)
Suppose sensor $S_i$ cannot distinguish $t$ and $t'$

\[
\begin{align*}
&d_{i,t} \quad d_{i,t'} + 2e \\
&\cos = \frac{d_{i,t}^2 + (4e)^2}{2d_{i,t} \cdot 4e} \quad \frac{d_{i,t'}^2}{2} \quad \frac{1}{2}
\end{align*}
\]

Then will be larger than $\frac{1}{3}$
Sufficient condition for localization coverage

- Sensor $S_1$ can not distinguish point $t$ and $t'$, but sensor $S_2$ can.
- If there are any sensor in the white area, as a $\frac{2}{3}$ sector, the two point can be distinguished.
Disk coverage model

- Ensure at least one sensor in every disk with radius of $0.464r$ ($r$ is the detection range)
- In other words, need more than 4 times sensor for coverage than detection

Benefits
- Disk model is well studied
- Can use simple algorithms to check coverage

Drawbacks
- Overestimates the density
Why select network resolution as 4e

- Network resolution can be selected as $\alpha e$
- We need extremely high density to guarantee a small $\alpha$
- Even if we select $\alpha$ as much larger than 4, we still need nearly same density as $\alpha=4$

Relationship between density and network resolution
Sector coverage

- Disk model is too strict and overestimates the density

- Sector coverage:
  -- approximate the void as two opposite sectors
  -- meet the coverage condition by ensuring nonexistence of such sector shaped void
Estimating density through sector coverage

- Assume sensors are uniformly distributed (Poisson Point Process)
- Given the density of sensors, estimate the probability that there are two opposite sector void around one point
- Provides more accurate density estimations, require 2 times less sensor than disk model
- More complex than disk model
Simulation results

Estimation of vacancy through disk and sector coverage
Simulation results

Vacancy size with different network resolution requirements
Conclusion

- Disk coverage is insufficient for some applications

- We show the sufficient condition for coverage of localization

- Two methods for estimating the necessary density for localization
  
  - **Disk coverage:** Require nearly 4 time more sensors than detection, low computational complexity
  
  - **Sector coverage:** More accurate estimation, use 2 times less sensors, require complex algorithms
Thank you!
Supplementary slides
Estimate density through sector coverage

The probability for there are $k$ sensors within the detection range

$$P_k = e^{-r^2} \left( \frac{r^2}{k!} \right)^k$$

The probability that there are two $2\pi/3$ sector voids given there are $k$ sensors around

-- $k=0,1$: probability=1
-- $k=2$: probability=2/3
-- $k>2$: probability=exist one $4\pi/3$ sector void
Probability of $4\pi/3$ sector void when $k$-covered

$Q_k$: All the $k$ sensors in a $2\pi/3$ sector not crossing the zero angle

$Q'_k$: All the $k$ sensors in a $2\pi/3$ sector crossing the zero angle

$$Q_k = k \left( \frac{1}{3} \right)^k (k - 1) \left( \frac{1}{3} \right)^k$$

$$Q'_k = \frac{1}{2} Q_k$$
Estimate density through sector coverage

- Overall the probability that a point is uncovered in sector model is bounded by

\[ P_0 + P_1 + \frac{2P_2}{3} + \sum_{k=3}^{\infty} Q_k P_K \]

- The probability that a point is uncovered in disk model

\[ P_{\text{disk}} = e^{-(0.464r)^2} \]