Coverage for Target Localization in Wireless Sensor Networks

Wei Wang, Vikram Srinivasan, Bang Wang, and Kee-Chaing Chua

Abstract— **Target tracking and localization are important applications in wireless sensor networks. Although the coverage problem for target detection has been intensively studied, few consider the coverage problem from the perspective of target localization. In this paper, we propose two methods to estimate the lower bound of sensor density to guarantee a bounded localization error over the sensing field. We first convert the coverage problem for localization to a conventional disk coverage problem, where the sensing area is a disk centered at the sensor. Our results show that the disk coverage model requires 4 times more sensors for *localization* compared to *detection* applications. We then introduce the idea of sector coverage to tighten the lower bound. The lower bound derived through sector coverage is 2 times less than through disk coverage. A distributed sector coverage algorithm is then proposed in this paper. Compared to disk coverage, sector coverage requires more computations. However, it provides more accurate density estimations than the disk model. Numerical evaluations show that the density bound derived through our sector coverage model is tight.**

Index Terms— **Sensor networks, coverage, localization.**

I. Introduction

ONE of the fundamental tasks for Wireless Sensor Networks is to collect information from the physical world. A network designer faces several challenges when designing a sensor network. Apart from the wireless medium, the primary challenges for sensor networks stem from two facts. First, sensors are extremely resource constrained. Second, in many applications sensor nodes will be randomly deployed. This randomness raises the issue of self-organization and equally importantly that of dimensioning the network. Scattering too few nodes may result in lack of coverage of the sensor field and a disconnected network. Scattering too many nodes may result in an inefficient network due to increased MAC collisions and interference. It is this critical aspect of dimensioning the network that we address in this paper.

The dimensioning problem from the point of view of *coverage* and *connectivity* has been intensely studied in recent years [2], [3]. The most commonly used model in coverage problems is the disk model, which assumes that the sensing region for a sensor is a circular region centered at it. A point is said to be covered by the sensor if it is within its sensing region. The disk model has certain limitations in describing how well the field is covered. First, it does not consider cooperative detection by multiple sensors. When several nearby sensors are monitoring an event at the same time, the estimation error can be reduced through cooperative signal processing [4]. Although each single sensor may not be able to provide precise information about the event, the information about the environment can still be reconstructed when the measurements from multiple sensors are combined. Thus, the sensing region for a cluster of sensors can be much greater than the union of their sensing disks. Second, the disk model is inadequate for certain applications. For example, when the objective of coverage is to localize a target within a certain error margin, ensuring that the region is covered by sensing disks cannot guarantee precise localization. Even with the concept of k-*coverage*, where every point should be covered by at least k sensors [5], the network still cannot provide a tight localization bound. As a counter example, consider the case where a target is covered by k sensors which are very close to each other. Although this target is k-covered, it cannot be precisely localized since the k sensors only provide redundant location information about the target.

In this paper, we investigate the coverage problem for an important sensor network application – target tracking and localization. We first define a *coverage hole* for localization as an area where the network cannot localize a target within a predefined error bound. We then use the concept of network resolution on localization to bound the localization error over the network. We show that we can convert requirements on localization error to corresponding requirements on network resolution. We then demonstrate how the network resolution is directly controlled by the density of sensors. Thus, we can estimate the density requirements of a localization application through network resolution.

We propose two methods to derive the relationship between sensor density and network resolution. The first method is based on a sufficient condition on node distribution which guarantees a lower bound of resolution over the whole field. Using this sufficient condition, the problem can be converted to a conventional disk coverage problem. Under the disk coverage model, we show that we need 4.64 times more sensors for localization than detection. However, this method may overestimate the sensor density required by localization. In order to tighten the bound on the necessary density, we introduce the idea of sector coverage. For the sector coverage model, we only need half the sensor density required by the disk coverage model. We then propose a distributed algorithm

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for sector coverage and show its computation complexity. Compared to disk coverage, which can utilize many existing efficient coverage algorithms, a sector coverage algorithm requires more computations. So, the two methods have different merits. The disk model provides a simple, but not very accurate way to estimate the sensor density, while the sector coverage model is more complex and accurate. Our numerical results are close to our analysis on the sector coverage model, proving the usefulness of our analytical methodology.

The rest of this paper is organized as follows: Section II summarizes previous work on coverage and localization problems. The system model and the definition of coverage in localization applications are described in Section III. Section IV gives a sufficient condition to provide bounded localization error and further relates this condition with the disk coverage model. Section V gives a better sensor density estimation method based on the idea of sector coverage. We then propose a distributed sector coverage algorithm in Section VI. Numerical results are provided in Section VII. Finally, Section VIII concludes the paper.

II. Related Work

The coverage problem in wireless sensor networks has been intensively studied in recent years [2], [3]. Most of these works look at the problem of covering every point in the sensing field with sensing disks [5], [6], [7] or detecting a target when it passes through the sensing field [8], [9]. In this paper, we investigate the coverage problem from a different view point by addressing the localization error of the sensor network. The assumption that the sensing region are disks no longer holds in the context of localization. However, we show that the results and algorithms in the previous works can also apply to the disk coverage model derived in Section IV. Our sector coverage concept is similar to the one studied in [10]. However, [10] focuses on network connectivity rather than localization.

Localization is an important component in sensor networks, since it provides coordinates both for the sensors [11] and for the targets in the sensor network [12]. In this paper, we focus on the problem where the sensor locations are already known and the objective is to localize targets in the sensing field. We use range based sensors to localize the target. Several well known estimation methods, such as Cramér-Rao Bound (CRB) [13] or Bayesian Bound (BB) [14], are used in range based localization systems. In this paper, we use a simplified localization model for the coverage problem. We try to bound the location estimation error to be within a small circle of radius ε with high probability. This is different from the normal approach of minimizing the mean squared value of the localization error. Our model is more suitable for systems where the performance is determined by the maximal localization error.

Connecting the coverage problem with the localization problem will provide useful guidelines for sensor network deployment [15]. Beacon deployment for sensor localization is studied in [16], [17]. Nagpal et al. point out that there should be voids around areas where the localization error is large [18]. In [18], the localization error is bounded through intersecting concentric rings centered at different beacons with a width same as the communication range. We use a similar method to bound the localization error by studying the intersection area of rings with width of $2e$ in this paper. However, we consider target localization rather than localization for sensors and we provide deeper analysis for range based localization systems.

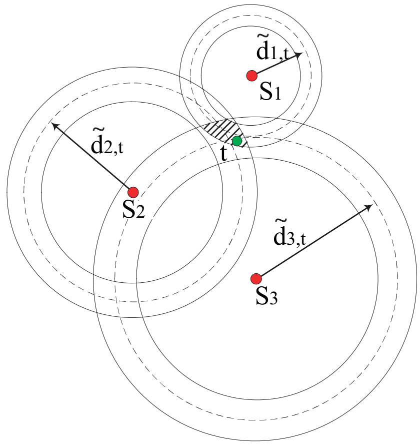

Fig. 1. Each of the three sensors S_1, S_2 and S_3 can provide distance information of the target at point t. The target location will be within the intersection of the annuli around them, shown as the shaded area.

III. System Model

We assume that the sensor network is running a target localization application, where the objective of the network is to provide accurate location information of the target. We assume that the coordinates of sensors are known, and the location of the target is estimated based on the measurements and coordinates of nearby sensors. We focus on networks formed by sensors which can measure their distance to the target, e.g., Time Difference Of Arrival (TDOA) and Received Signal Strength Indicator (RSSI) sensors.

Due to the existence of noise, the distance estimation will be distributed within a certain range around the true distance. When the true distance between sensor S_i and a target at point t is $d_{i,t}$, we assume the estimated distance $\widetilde{d_{i,t}}$ by sensor i will fall in the range $[d_{i,t} - e_l, d_{i,t} + e_u]$ with high probability, where e_l and e_u are the error bounds. We set e_l, e_u equal to e in the latter derivation. However, our result can be easily extended to the case where e_l and e_u are not equal. We assume that sensors can only provide distance measurements when the target is within its detection range of r. We define the *normalized measurement accuracy* of a sensor as $K = r/e$, which is the ratio of the detection range divided by the maximum measurement error within the detection range. Under the assumptions we make, when sensor S_i gives a distance estimate of $\widetilde{d_{i,t}}$, the target will fall in $[\widetilde{d_{i,t}} - e, \widetilde{d_{i,t}} + e]$ with high probability. Then, a single sensor i can localize the target within an annulus of $[\widetilde{d_{i,t}} - e, \widetilde{d_{i,t}} + e]$, as shown in Fig. 1.

When the target can be detected by several sensors at the same time, the position estimation can be further refined by cooperative signal processing. In this paper, we combine the sensor measurements by intersecting the annuli of different sensors [19], as shown in Fig. 1. The intersection area is defined as the *uncertainty region* of localization. As more sensors provide distance information about the target, the uncertainty region will become smaller. Given the group of distance measurements $\widetilde{d_{1,t}}, \widetilde{d_{2,t}}, \ldots, \widetilde{d_{k,t}}$ provided by all the k sensors which can detect the target, we can determine the smallest uncertainty region. The estimation of the target location can be set as the center of the smallest circle which can circumscribe this smallest uncertainty region. The localization error is smaller than the radius of this circle, since the distance from all points in the uncertainty region to the center is smaller than the radius. Note that the uncertainty region may contain multiple disconnected intersection areas. We use the union of such area as the uncertainty region so that it includes all possible target positions. If the sensor density is high enough, we can make all uncertainty regions small enough to be well contained in circles with radius smaller than ε. In this case, the localization error is always smaller than ε, no matter where the target is.

Although our localization method is much simpler compared to currently used localization algorithms, this simplified model retains the basic ideas of range based localization while at the same time revealing key insights to relationships between the coverage and localization problem. We also do not use time-correlated measurements for localization, i.e., sensors may use the measurements of previous target positions to infer current target location. Although localization accuracy can be improved by time-correlated localization, such methods are highly dependent on the assumptions of target movement speed and sensor measurement intervals. In this paper, we focus on localization with measurements taken at the same time and the sensor density derived here can serve as an upper bound for time-correlated localization.

The concept of coverage used in this paper is defined as follows. For the target localization application, we require the location estimation error to be within a circle of radius ε. In a randomly deployed sensor network, there may exist areas where the local density is so low that we cannot precisely localize the target. The *coverage hole* is defined as a hole where the localization error exceeds the predefined bound. The objective of this paper is to find the sensor density which can ensure that there is no coverage hole in the network or the ratio of coverage hole compared to the total area of the field is below a given bound.

IV. Coverage in Target Localization Applications

A. *Sufficient Condition for Coverage*

In this section, we give a sufficient condition for the localization error to be bounded over the whole field. We first introduce the concept of network resolution on localization. We say that two points t and t' are distinguishable if the sensor network can always distinguish whether a target is at point t or at point t' through the distance measurements provided by sensors. The *network resolution* is the minimum distance l, such that the network can distinguish any pair of points when the distance between them is larger than l. The network resolution is related to the localization error bound ε by the following lemma.

Lemma 1: Given the network resolution of l, the localization estimation error ε is upper bounded by $l/\sqrt{3}$ and lower bounded by $l/2$.

Proof: We see that points in the same uncertainty region cannot be distinguished by the network. If the network resolution is l, then every uncertainty region should not contain two points apart by more than l. Thus, the *Generalized Diameter*, which is defined as the greatest distance between any two points in the shape, is smaller or equal to l for all uncertainty regions. A hexagon with side length of $l/\sqrt{3}$ can fully cover any shape with Generalized Diameter smaller than l [20]. Thus, the circumcircle of such a hexagon, which has a radius of $l/\sqrt{3}$, can also cover any uncertainty region in the network. Using the center of the covering circle as the estimated location of the target will provide estimation error smaller than $l/\sqrt{3}$. This bound is tight since when uncertainty region is shaped as an equilateral triangle with side length of l, the smallest circle which can circumscribe it has radius of exactly $l/\sqrt{3}$.

If the network resolution is l, then there should exist two points in an uncertainty region that are apart by l by definition. Such an uncertainty region cannot be circumscribed by circles with radius smaller than $l/2$. Thus, the estimation error is larger than $l/2$. ■

Lemma 2: For two points t and t', if there exists a sensor S_i whose distance to these two points satisfies:

$$|d_{i,t} - d_{i,t'}| \geq 2e \tag{1}$$

then sensor S_i can definitely distinguish point t from point t', given that at least one of these two points is in the detection range of S_i ($d_{i,t} < r$ or $d_{i,t'} < r$).

Proof: Without loss of generality, suppose the true target position is at t. According to the assumption of distance measurement error, the distance measurement $\widetilde{d_{i,t}}$ provided by sensor S_i should satisfy:

$$|\widetilde{d_{i,t}} - d_{i,t}| \leq e \tag{2}$$

Since $|d_{i,t} - d_{i,t'}| \geq 2e$, we have:

$$|d_{i,t'} - \widetilde{d_{i,t}}| \geq |d_{i,t'} - d_{i,t}| - |d_{i,t} - \widetilde{d_{i,t}}| \geq e \tag{3}$$

Thus, the point t' will be outside the uncertainty annulus of $[\widetilde{d_{i,t}} - e, \widetilde{d_{i,t}} + e]$ and sensor S_i can determine that the target is not at point t' based on distance measurement of $\widetilde{d_{i,t}}$. ■

Theorem 1: If there is at least one sensor in any arbitrarily selected sector of radius r (the predefined detection range) and angle $\frac{2\pi}{3}$, denoted as a sector of $(r, \frac{2\pi}{3})$, then the location estimation error is bounded by $\varepsilon = \frac{4\sqrt{3}e}{3}$ over the network, where e is the maximal distance estimation error when the target is within the detection range r.

Proof: As shown in Lemma 1, if the network resolution $l \leq \sqrt{3}\varepsilon = 4e$, we can guarantee a location estimation error bound of $\varepsilon = \frac{4\sqrt{3}e}{3}$. We will prove this by contradiction. Suppose the network resolution is worse than $4e$, which means we can find at least one pair of points t and t' which are apart by more than $4e$, but no sensor can distinguish them.

Suppose points t and t' cannot be distinguished by the network, and the distance between t and t' is $4e$. As we

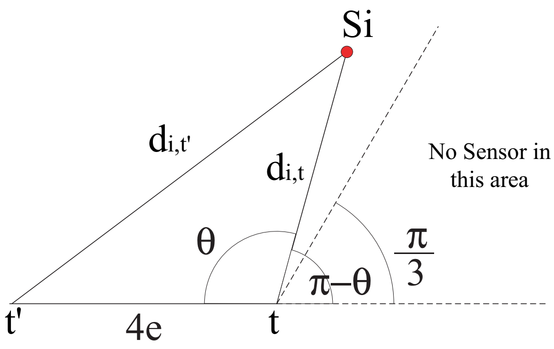

Fig. 2. Sensor S_i can detect both point t and t', but cannot distinguish them.

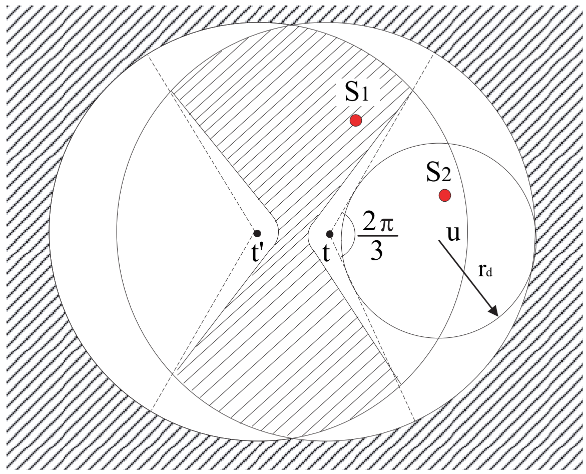

Fig. 3. Sensors in the shaded area, such as S_1, cannot distinguish the point t from t'. Sensors in the white area, such as S_2, can distinguish these two points, so there should not be any sensor in the white area.

assumed, there is at least one sensor in every arbitrarily selected sector of $(r, \frac{2\pi}{3})$. So, there must be at least one sensor within range r of point t. Suppose this sensor is sensor S_i. Then sensor S_i should be able to detect the point t', otherwise these two points can not be in the same uncertainty region and the network can distinguish them. Without loss of generality, suppose sensor S_i is closer to the point t, i.e., $d_{i,t} < d_{i,t'}$, as shown in Fig. 2. From Lemma 2, we have $d_{i,t'} \leq d_{i,t} + 2e$ when S_i can not distinguish t and t', then:

$$\cos\theta = \frac{d_{i,t}^2 + (4e)^2 - d_{i,t'}^2}{2 \times d_{i,t} \times 4e} \geq \frac{12e^2 - 4d_{i,t}e}{8d_{i,t}e} \geq -\frac{1}{2} \quad (4)$$

Since $\cos\theta$ is monotonically decreasing in $[0, \pi]$, the angle θ must be smaller than $\arccos(-\frac{1}{2}) = \frac{2\pi}{3}$, so the angle $\pi - \theta$ is larger than $\frac{\pi}{3}$. By symmetry, no sensor can be within the sector of $(r, \frac{2\pi}{3})$ centered at t and bisected by ray $t't$. This is a contradiction to the assumption that there should be at least one sensor in any arbitrarily selected sector of $(r, \frac{2\pi}{3})$.

Eq. (4) considers the case that $d_{t,t'} = 4e$. If $d_{t,t'} > 4e$, this lower bound for $\pi - \theta$ increases monotonically. Thus, if the two indistinguishable points t and t' are separated by more than $4e$, there also should be no sensor in the sector of $(r, \frac{2\pi}{3})$ centered at t, which contradicts our assumption. Therefore, there does not exist indistinguishable points t and t', which are apart by more than $4e$. Then, the network resolution is better than $4e$, which directly leads to a localization error bound of $\frac{4\sqrt{3}e}{3}$ by lemma 1. ■

Theorem 1 can be directly extended to the disk coverage model as follows.

Corollary 1: Given a sensor deployment, if disks of radius $\frac{\sqrt{3}}{\sqrt{3}+2}r$ centered at the sensors can cover the entire field, then the location estimation error is bounded by $\varepsilon = \frac{4\sqrt{3}e}{3}$ over the network.

Corollary 1 comes from the fact that a sector of $(r, \frac{2\pi}{3})$ contains an inscribed circle of radius $r_d = \frac{\sqrt{3}}{\sqrt{3}+2}r \approx 0.464r$, as shown in Fig 3[1]. If there exists a $(r, \frac{2\pi}{3})$ sector void, then there is no sensor within a radius of r_d from the center u of the inscribed circle. Therefore, if the sensor density is high enough that the field is totally covered by disks of radius r_d centered at sensors, then there will be no $(r, \frac{2\pi}{3})$ sector void and the location estimation error bound is guaranteed. Thus, we can directly use known results in disk coverage for localization applications by shrinking the radius of coverage disk to r_d.

Corollary 1 shows that we need to deploy sensors at a higher density to localize the target. We can estimate the density required for localization coverage as follows. Assume we have to provide coverage over a square field of area A. If we look at a square field with sides scaled by a factor of 0.464, the number of sensors required to cover this field with sensing radius r_d is the same as the number of sensors required to cover the area of A with radius r. However, the same number of sensors only cover an area of $0.464^2 A$ with a shrunk sensing radius of r_d. Therefore the sensor density required for localization coverage is $\frac{1}{0.464^2} \approx 4.64$ times more than that required for a detection coverage.

Using sensing range larger than r_d in the disk model may violate the conditions in Theorem 1. Consider the case where sensors are densely deployed on the border of the shaded area in Fig. 3. With sensing disks larger than r_d, we can get the area disk covered (or even k-covered). However, the conditions in Theorem 1 are violated. Those sensors in the shaded area can not distinguish point t from t' and the localization error can exceed the bound. Thus, shrinking the disk radius to r_d is necessary in disk model.

B. Relationship between Resolution and Density

Theorem 1 shows how to guarantee network resolution of $4e$. As the uncertainty region of a single sensor is an annulus with width of $2e$, we may further improve the network resolution to $2e$ by increasing the sensor density. However, we show below that this will not be cost efficient.

Suppose that we require a network resolution of $l = \alpha e$ with $\alpha \geq 2$, since the network resolution is lower bounded by the annulus width of $2e$ used in localization. Substituting this into Eq. (4), we have:

$$\cos\theta = \frac{d_{i,t}^2 + (\alpha e)^2 - d_{i,t'}^2}{2 \times d_{i,t} \times \alpha e}$$

[1]Note that Eq. (4) gives a lower bound of the void. The white area drawn here, which is derived by Eq. (1), is larger than a $(r, \frac{2\pi}{3})$ sector.

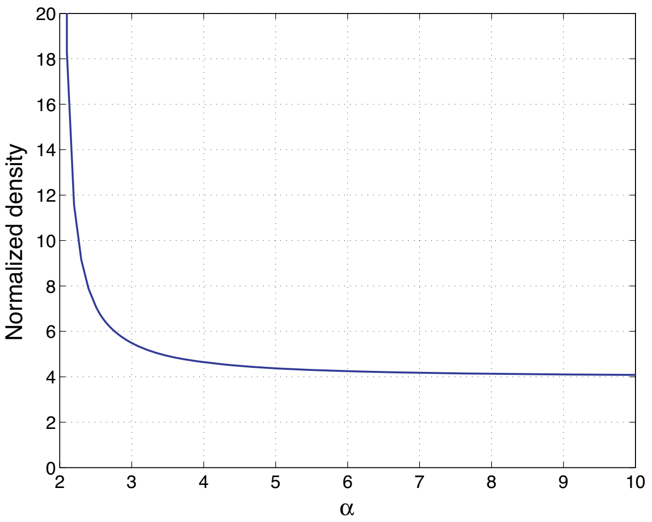

Fig. 4. The relationship between network resolution and sensor density. α is network resolution divided by e, the distance measurement error. The normalized density is ratio of localization coverage compared to detection coverage.

$$\geq \quad \frac{(\alpha e)^2 - 4d_{i,t}e - 4e^2}{2\alpha d_{i,t}e} \geq -\frac{2}{\alpha} \tag{5}$$

When $d_{i,t}/e$ is large, θ_{max} can be exactly equal to $\arccos(-\frac{2}{\alpha})$. As α decreases from 4 to 2, θ_{max} grows from $2\pi/3$ to π. As shown in the proof of Theorem 1, we require every sector of $(r, 2(\pi-\theta))$ to contain at least one sensor. For small values of α, the angle of such sectors decreases to zero; thus an extremely high density is required to have a network resolution close to $2e$.

To demonstrate this, we investigate the relationship between α and sensor density under the disk coverage model. The maximal radius for a circle which can be packed in the $2(\pi-\theta)$ sector is $\sin\theta/(1+\sin\theta)$. Based on this, the relationship between sensor density and network resolution is plotted in Fig. 4. When the resolution requirement is close to $2e$, sensor density increases quickly, yet the gain in network resolution is small. On the other hand, when network resolution is worse than $4e$, sensor density remains nearly constant and it converges to 4 times the detection density as α goes to infinity. The radius of the sensing disk is upper bounded by $0.5r$ when α is increased. This means that we need at least one sensor in each disk of radius $0.5r$ to meet the basic density requirement for localization, e.g., the target is detected by at least 3 sensors at appropriate positions for triangulation. Fig. 4 hints that a network resolution of $3e$ to $4e$ is the best trade-off between sensor density and network resolution. Therefore, when sensors with low accuracy are used, we cannot simply increase the sensor density to achieve high network resolution.

V. Density Estimation Through Sector Coverage

From the proof of Theorem 1, we can see that when the localization error exceeds the bound, there are voids shaped as the white area in Fig. 3. Showing that there is no disk shaped void of radius r_d in the network can eliminate the existence of such sector-shaped voids. However, a network may not contain such sector shaped voids even if it is not covered by disks of radius r_d. Thus, the sufficient condition derived by disk coverage is too strict. It may give a higher estimation on sensor density.

In this section, we introduce the concept of *sector coverage* to provide a better estimate on sensor density required for a randomly deployed network to guarantee a bounded localization error. The derivation is based on the assumption that sensors are randomly scattered in the field with distribution of a stationary Poisson point process with intensity λ [21]. We use the average vacancy over a unit area, which is often called porosity in coverage theory [21], to measure the quality of coverage. The average vacancy over a unit area is defined as the ratio of uncovered area divided by the total area of the field. It is equal to the probability that an arbitrary point is not covered, i.e., the probability that localization error exceeds the bound at that point.

Instead of estimating whether there are voids shaped as the white area in Fig. 3, we approximate the voids using two opposing sectors of $(r, \frac{2\pi}{3})$ around point t. We call this approximation *sector coverage*: a point is sector covered if there is at least one sensor in any pair of opposing sectors of $(r, \frac{2\pi}{3})$ around it in any arbitrary orientation. We need to check sectors in all orientations since the sufficient condition in Theorem 1 can be violated when there exists void in any orientation.

The approximation of sector coverage is based on the following observations. Suppose that t is uncovered. Then there will be a point t' which cannot be distinguished from point t and $d_{t,t'}$ is larger than $4e$. Following the arguments in Theorem 1, there should also be a sector void of $(r, \frac{2\pi}{3})$ around t' by symmetry. When the sensor accuracy K is large, the two white areas around t and t' can be treated as two opposing sectors of $(r, \frac{2\pi}{3})$ around point t.

The approximation can be broken into two steps. First, we approximate the two white areas around t and t' in Fig. 3 by two $(r, \frac{2\pi}{3})$ sectors at point t and t'. When K is large, we have $r \gg e$. The angle θ defined in Eq. (4) converges to $\frac{2\pi}{3}$ as $d_{i,t}$ approaches r, which means the border of the shaded area in Fig. 3 will overlap with the border of the sectors. Furthermore, as $d_{t,t'}$ is small compared to r, the two circles will almost overlap with each other. Therefore, the difference between the $(r, \frac{2\pi}{3})$ sector and the white area in Fig. 3 can be ignored in this case. Note that the white areas are strictly larger than the two $(r, \frac{2\pi}{3})$ sectors for all K values by Theorem 1. Therefore, when K is small, our approximation can serve as an upper bound for the probability that a point is not covered.

Second, we approximate the two sectors around t and t' as a pair of opposing sectors centered at point t. This approximation is valid since the sensors are distributed as a Poisson point process. In this case, the sensor distribution in disjoint areas are independent. Thus, the probability that there exist voids of a pair of opposing sectors centered at point t is same as the probability that there exist voids of two sectors which are apart by $d_{t,t'}$, given that the sector orientation and $d_{t,t'}$ are fixed [21]. Section VII will further validate our approximation by simulations.

We now proceed to find the density requirement for sector coverage. Consider a Poisson point process with intensity λ. The probability that there are k sensors in the detection range

r of point t is given by:

$$P_k = e^{-\pi\lambda r^2}\frac{(\pi\lambda r^2)^k}{k!} \tag{6}$$

For sensors falling within the range r, we denote the sensor's position as (a_i, ϕ_i) in polar coordinates with the origin at t. Using the property of Poisson process, the ϕ_is are independently and uniformly distributed over $[0, 2\pi]$ given there are k sensors in the circle.

If there is no more than one sensor in the detection range, we can always find two opposing $(r, \frac{2\pi}{3})$ sector voids, thus the point t cannot be sector covered in this case. If there are two sensors within range r and the angle between these two sensors falls in the range of $[0, \frac{\pi}{3}] \cup [\frac{2\pi}{3}, \pi]$, then the point t is not sector covered[2]. Since the angle $|\phi_1 - \phi_2|$ is uniformly distributed in $[0, \pi]$, the probability that a point t is not sector covered given there are two sensors in the detection range is $\frac{2}{3}$.

If there are $k > 2$ sensors in the circle, we first convert the problem of finding two opposing sector voids of $\frac{2\pi}{3}$ to finding one continuous sector void of $\frac{4\pi}{3}$. For every sensor, define ϕ'_i as

$$\phi'_i = \begin{cases} 2\phi_i, & 0 \le \phi_i < \pi, \quad i = 1, 2 \ldots, k \\ 2(\phi_i - \pi), & \pi \le \phi_i \le 2\pi, \quad i = 1, 2, \ldots k \end{cases} \tag{7}$$

Two opposing sensors with angle of ϕ_i and $\phi_i + \pi$ will be transformed to the same angle of $\phi'_i = 2\phi_i$. Thus, any sensor deployment that has a void of two opposing sector of $\frac{2\pi}{3}$ will have a void of $\frac{4\pi}{3}$ in the transformed coordinate system of (a_i, ϕ'_i). It is easy to see that the angle ϕ'_i is also independently and uniformly distributed on $[0, 2\pi]$.

For k independently and uniformly distributed ϕ'_i, the probability that the range of the samples, defined as $\max\{\phi'_i\} - \min\{\phi'_i\}$, is smaller than $\frac{2\pi}{3}$ is given by [22]:

$$Q_k = k\Big(\frac{1}{3}\Big)^{k-1} - (k-1)\Big(\frac{1}{3}\Big)^k \tag{8}$$

Q_k is the probability that all k sensors are confined in a $\frac{2\pi}{3}$ sector with the remaining $\frac{4\pi}{3}$ sector as a void. Since sectors can cross the zero angle, Q_k does not account for the case that all k sensors are confined in a sector with an angle smaller than $\frac{2\pi}{3}$ and it crosses the zero angle (see Fig. 5). We define the probability of this scenario as Q'_k. It is clear that Q'_k is smaller than the probability that k sensors are confined in a $(r, \frac{2\pi}{3})$ sector *and* the sector starts from $[\frac{4\pi}{3}, 2\pi]$. This probability is half the probability that sensors are confined in a $(r, \frac{2\pi}{3})$ sector *and* the sector starts from $[0, \frac{4\pi}{3}]$ due to the uniform distribution of ϕ'_i. Since Q_k includes all cases that sensors are confined in a $(r, \frac{2\pi}{3})$ sector *and* the sector starts from $[0, \frac{4\pi}{3}]$, Q'_k is upper bounded by $Q_k/2$. Combining all these cases, the probability that an arbitrary point t is not sector covered is upper bounded by:

$$P_{sector} \le P_0 + P_1 + \frac{2P_2}{3} + \frac{3}{2}\sum_{k=3}^{\infty} Q_k P_k \tag{9}$$

[2]Note that sector coverage is an approximation. A point can be sector covered when there are only two sensors in the detection range, yet the localization error may exceed the bound.

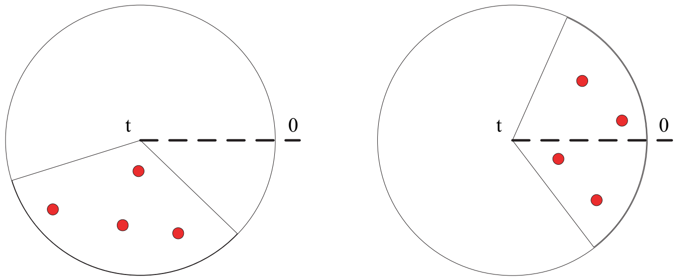

Fig. 5. Bounding the probability that there exists a sector void of $(r, \frac{2\pi}{3})$, two scenarios are considered based on whether the sector crosses the zero angle.

For disk coverage, the probability that one point is not covered is given by:

$$P_{disk} = e^{-\pi\lambda r_d^2} \tag{10}$$

which is the chance that there is no sensor in the disk of radius r_d. For a given coverage requirement of average vacancy v over a unit area, we can calculate the necessary density for both sector coverage and disk coverage, denoted as λ_{disk} and λ_{sector}.

Fig. 6 shows that the density requirement for sector coverage is much smaller than the disk model. For sector coverage, the density requirement for localization is nearly 1.8 times higher than the density required for detection coverage when v is small. As v becomes smaller, $\lambda_{disk}/\lambda_{sector}$ converges to 2.5, which means sector coverage requires 2.5 times fewer sensors than the equivalent disk coverage. This indicates that disk coverage is not accurate when used for localization applications.

Another important observation is that the value of $\lambda_{disk}/\lambda_{sector}$ is not a constant. This means the sector coverage cannot be approximated by fixed shapes, e.g., two opposite sectors of $(r, \frac{2\pi}{3})$ or a pair of circles of radius r_d with fixed orientations. In that case, if the area of the shape is A_s, the average vacancy will be $e^{-\lambda A_s}$ [21]. So, the required density will simply be $\frac{A_s}{\pi r_d^2}\lambda_{disk}$, which is λ_{disk} multiplied by a constant.

VI. Distributed Sector Coverage Algorithms

In high density wireless sensor networks, sensors need to periodically switch to the sleep state to save energy [23]. To ensure that the field is well monitored, the awake sensors should provide coverage over the whole field. In other words, a sensor can only go to sleep when there is no coverage hole in its sensing area. Thus, we often need a distributed coverage algorithm to check whether there are coverage holes in the field when the position of the waking sensors are known.

For the disk coverage model, a sensor can determine whether its sensing region is k-covered by checking the intersection points of the sensing boundary of its neighbors [24]. This gives an algorithm of complexity $O(n^3)$, where n is the number of waking neighbors within range of $2r$. However, in the sector coverage model, a sensor needs to check whether there are sector voids around it, which requires more computation.

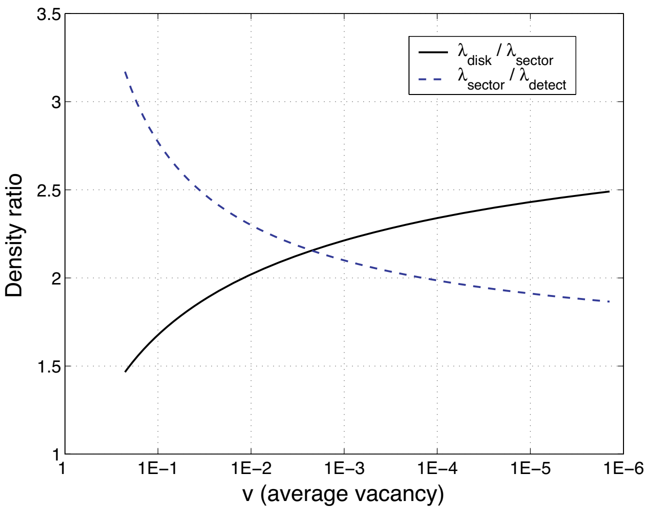

Fig. 6. Comparing the density requirements for disk coverage and sector coverage.

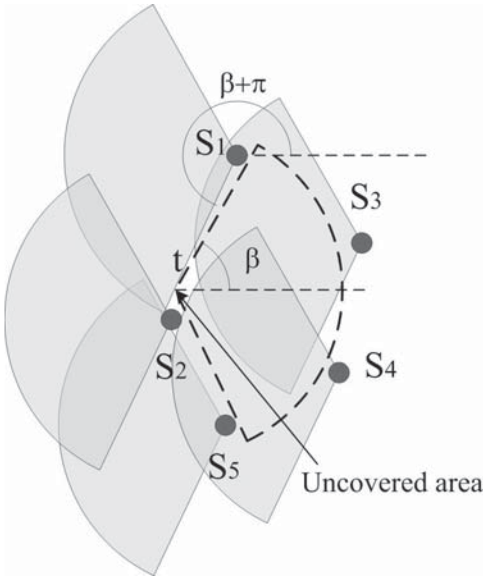

Fig. 7. Sector coverage for a given orientation of β.

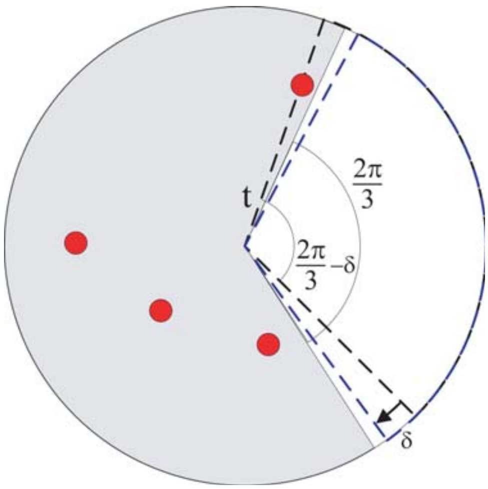

Fig. 8. Increasing β by step size of δ. The void is two opposite sectors of $(r, \frac{2\pi}{3})$, as the two white sector in the figure. The sensing region is the dotted sectors, which is $(r, \frac{2\pi}{3} - \delta)$.

A. Sector Coverage

As in the classical coverage theory for arbitrary shapes, we define a point set $C \subseteq \mathbb{R}^2$ as the sensing region of one sensor [21]. The *Minkowski sum* of a point $\{x\}$ and C is defined as $\{x\} + C \equiv \{x + y : y \in C\}$, which is a translation of C by $\{x\}$.

For a field with sensors deployed at points $S_1, S_2, \ldots, S_n$, a point t is said to be not covered when: $t \notin \{S_i\} + C, \forall i$. Conversely, we have $S_i \notin \{t\} + C^*, \forall i$, where $C^* = \{-y : y \in C\}$. For example, we have $C = \{x \in \mathbb{R}^2 : |x| < r\}$ for the disk model. Then, a point t is said to be not covered when it is not in any circle with radius r centered at a sensor. Also, there will not be any sensor in the circle of radius r centered at the uncovered point t.

By Theorem 1, the network resolution can be guaranteed when there are no sector voids of $(r, \frac{2\pi}{3})$ in the network. Consider a particular point t in the network which has a sector void of $(r, \frac{2\pi}{3})$ with orientation of β around it, see Fig. 7. Accordingly, we have the sensing regions C^* as sectors with orientation of $\beta+\pi$ and point t will lie in the uncovered area in this case, see Fig. 7. Therefore, if the field can be fully covered by $(r, \frac{2\pi}{3})$ sectors with orientation of $\beta + \pi$, then there will be no sector void with orientation of β in the field. Since a point is sector covered only if it is covered for all orientations, we need to exhaustively check for all $\beta \in [0, 2\pi]$, which is practically impossible.

B. Simplified Sector Coverage Algorithm

There are several ways to reduce the computational complexity of sector coverage algorithms.

First, we can only check for discrete orientations of β. Instead of using a sector of $(r, \frac{2\pi}{3})$ as the sensing region, we can use a sector of $(r, \frac{2\pi}{3} - \delta)$, where δ is some small value. Then, we can increase β from 0 to 2π with step size of δ. If there is a sector void of $(r, \frac{2\pi}{3})$ around point t for a certain orientation, there always exist a certain step that the $(r, \frac{2\pi}{3} - \delta)$ sector is fully included in it when the step size is δ, as shown in Fig. 8. Thus, we can definitely detect the void using step size of δ and sensing region of a $(r, \frac{2\pi}{3} - \delta)$ sector. However, this scheme may mistake a point as an uncovered point when there are only voids of $(r, \frac{2\pi}{3} - \delta)$ sectors around it. The network resolution at the point will actually be better than the desired value of $4e$, but it can still be taken as uncovered.

Using Eq. (5), the network resolution at a point is $l = \alpha e$, where $\alpha = -\frac{2}{\cos\theta}$. For a point with a void of $(r, \frac{2\pi}{3} - \delta)$ sector, we can easily get $\theta = \frac{2\pi}{3} + \frac{\delta}{2}$. Thus, the network resolution is $l = -\frac{2e}{\cos(\frac{2\pi}{3} + \frac{\delta}{2})}$. When δ is small, this can be approximated by Taylor expansion as: $l = (4 - 2\sqrt{3}\delta)e$. For example, when $\delta = 0.01\pi$ (check for 200 different β values), the lower bound of network resolution around an uncovered point detected by the approximate scheme will be $3.89e$, which is close to $4e$. Also, for all points with network resolution larger than $4e$, our scheme can always detect it. This shows our discrete scheme is a good approximation to the exhaustive checking scheme. By choosing the step size δ, we can control the approximation ratio as required by the application.

For a fixed orientation, the sector coverage can be checked using a similar method as in the disk coverage [24]. If all intersection points of sectors of different sensors can be covered, then the field is fully covered. For the sector-shaped sensing region with the same orientation, the border of the sensing region of two sensors can intersect at most on 4 different points[3]. Calculating the intersection points for two sensors can be done in constant time. When there are n sensors within distance of $2r$, there are $O(n^2)$ intersection points to be checked. For each intersection point, we need to check whether it is within the sensing region of the other $n-1$ sensors. This gives computation complexity of $O(n^3)$, which is similar to the disk coverage. However, we need to repeat this algorithm $\frac{2\pi}{\delta}$ times. The overall computation complexity for our approximate sector coverage algorithm is $O(\frac{n^3}{\delta})$.

We can use the disk coverage model to exclude certain scenarios before using the sector coverage algorithm. As we have shown in Section V, a point can only be sector covered when it is disk covered by more than 2 sensors. So, if a sensor's sensing disk is not 2-covered by other sensors, it cannot go to sleep. Furthermore, if there are sector voids of $(r, \frac{2\pi}{3})$, there would be voids of disks with radius r_d. If shutting down a sensor S_i will generate a new sector void, the center u of an inscribed circle of the sector containing S_i will be an uncovered point when the sensing region are disks with radius r_d. Any point in the sector of $(r, \frac{2\pi}{3})$ will be at most $\sqrt{r_d^2 + (r - r_d/\sqrt{3})^2} \approx 0.867r$ away from the point u. Then, if there are no voids of disk coverage of range r_d within distance of $0.867r$ to the sensor S_i, we can safely shut down sensor S_i without generating new sector voids.

In summary, we list the steps for a sensor S_i to check whether it can go to sleep:

1. First, the sensor needs to know the position of all waking sensors within $2r$ distance from it.

2. If the disk with radius $0.867r$ around it can be fully covered by disks of radius r_d centered at other waking sensors, it can go to sleep and the algorithm terminates.

3. If the disk with radius r around it cannot be fully 2-covered by disks of radius r centered at other waking sensors, it cannot go to sleep and the algorithm terminates.

4. For each β from 0 to 2π with step δ, check if the disk with radius r around it can be fully covered by sectors of $(r, \frac{2\pi}{3} - \delta)$ with orientation of $\beta + \pi$ centered at other waking sensors. If there is any such void, it cannot go to sleep.

Compared to disk coverage algorithms, the sector coverage algorithm has computation complexity of $O(\frac{n^3}{\delta})$ rather than $O(n^3)$. So, sector coverage requires more computation when δ is small. The communication overhead of sector coverage is the same as the disk coverage. Our sector coverage algorithm only needs to know the information of neighboring sensors within range of $2r$ (their location and sleep-wake state). Such information is also required by disk coverage algorithms [24]. So, the information exchange protocols for disk coverage can also be used in our sector coverage algorithm. Note that in this algorithm we check for voids of single sector to guarantee network resolution. We can also use a similar algorithm to check whether there are two opposite sector voids of $(r, \frac{2\pi}{3})$ in the network. Such algorithm can use fewer sensors to cover the network, while it cannot provide guarantees on the network resolution.

[3]If a line segment of a sector overlaps with border of other sectors, it is enough to check only the end points of the line segment.

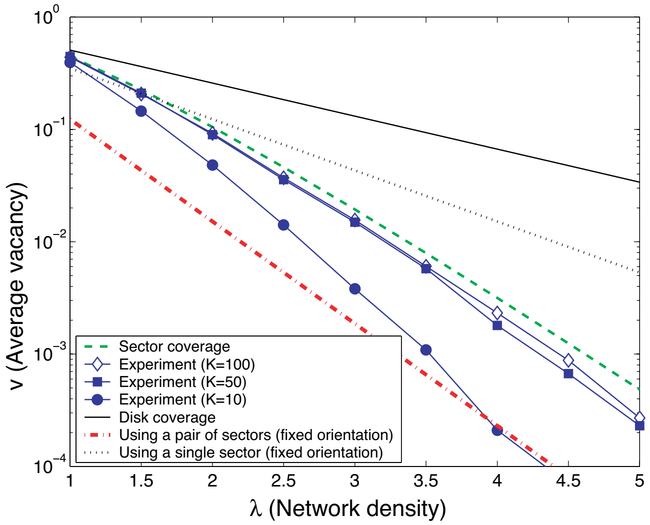

Fig. 9. Experimental results on average vacancy for different sensor accuracy.

VII. Numerical Evaluations

A. Experimental Setting

We use Monte Carlo methods to verify our theoretical analysis. The experimental setting is as follows: We randomly deploy λA sensors in a large region with area A, so we approximately get a Poisson point process with intensity λ in a small area inside A. We check whether there exists a pair of points which cannot be distinguished by the randomly deployed sensors. The minimum distance between two points which cannot be distinguished will be the experimental network resolution. We repeat over 100,000 network instances to obtain the probability that the network resolution is worse than αe.

B. Average Vacancy

The average vacancy is the probability that the network resolution is worse than the predefined bound. The experimental average vacancy is obtained through counting the vacancy probability at randomly picked point in different network instances. The result is shown in Fig. 9. The sector coverage model provides good estimates for the average vacancy when the sensor accuracy K, is larger than 50. For small K values, the white area in Fig. 3 is much larger compared to the $(r, \frac{2\pi}{3})$ sector approximation we used in the sector coverage, so it is much easier to get a point to be covered. Thus, the average vacancy is smaller than the estimated value. The vacancy estimation by the disk coverage method is much larger than the experimental result, which means the disk model is not accurate in estimating the vacancy. Sector coverage provides better estimates, however it is more complex for a sensor to determine whether the area around it is sector covered.

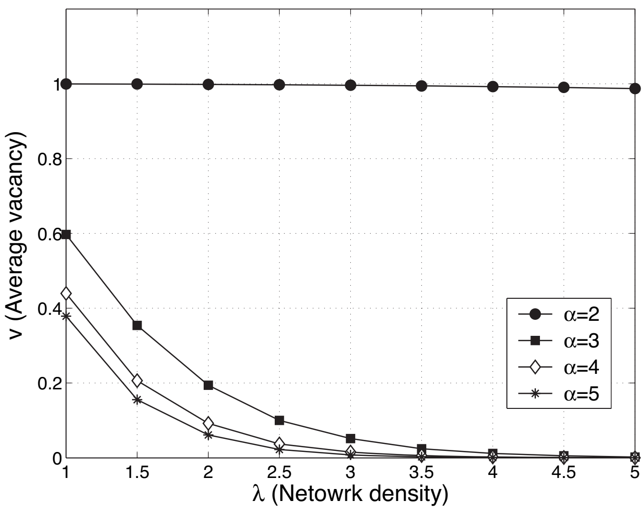

Fig. 10. Experimental results on average vacancy for different network resolution ($K = 100$).

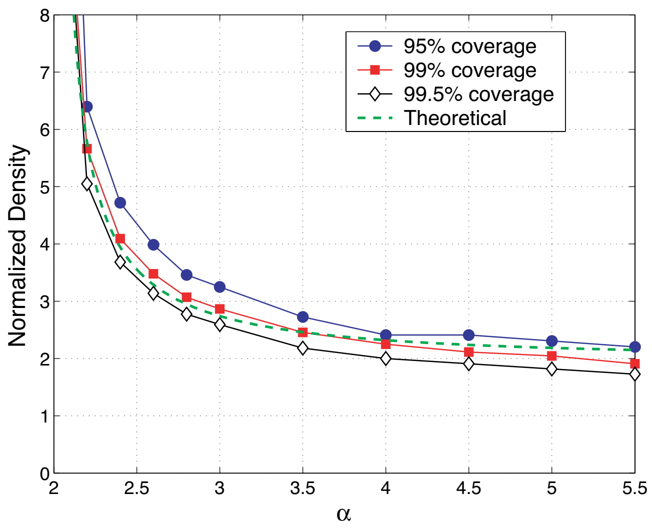

Fig. 11. Experimental results of necessary network density with different average vacancy requirements ($K = 100$). Normalized density is the ratio of density compared to the detection coverage density.

We also compare the sector coverage with estimations based on sectors with *fixed* orientations. In this case, the average vacancy estimation will be $e^{-2\pi\lambda r^2/3}$ and $e^{-\pi\lambda r^2/3}$ when using two opposite $(r, \frac{2\pi}{3})$ sectors and a single $(r, \frac{2\pi}{3})$ sector, respectively. We see that checking for all orientations greatly improves the estimation accuracy.

C. Average Vacancy with Different α

Recall that we can represent the network resolution as αe, where e is the distance estimation error. Fig. 10 shows the average vacancy under different α values. For $\alpha = 2$, more than 95% of the network area remain as vacancies even when the network density is high. This verifies our theoretical results in Section IV-B, which shows it is difficult to get localization error of $2e$ by increasing sensor density. For $\alpha > 3$, the average vacancy decreases sharply as the sensor density increases. For larger α, the average vacancy decreases faster as the network density increases. However, the difference becomes smaller when α is large. The curve for $\alpha = 4$ is very close to the one for $\alpha = 5$, which means achieving $\alpha = 4$ requires almost the same density for $\alpha = 5$. This result agrees with our theoretical analysis in Section IV-B.

D. Network Resolution

Fig. 11 shows the simulated relationship between the network density and network resolution. In the simulation, we find the smallest network density which can guarantee that at least a given coverage ratio, say 99%, of the whole field can get a resolution smaller than αe. In Section IV-B, we derive this relationship based on the disk coverage model. As we know that the disk coverage model may over-estimate the network density by about 2-2.5 times, we need to verify whether this overestimation will change the trend shown in Fig. 4. Since Fig. 6 shows $\lambda_{disk}/\lambda_{sector}$ is about 2 when average vacancy is around 1%, we divide the density derived through disk coverage (as in Fig. 4) by two and set this curve as our theoretical value. Comparing the simulation results and the theoretical value, we see that the relationship derived through disk coverage is quite accurate, except the density is halved due to the difference between λ_{disk} and λ_{sector}. The simulation results show that the curve in Fig. 4 truly reflects the relationship between the network density and the network resolution and a network resolution of $3e$ to $4e$ is still a good trade-off as we expected. We also see that the normalized density decreases as the coverage ratio becomes higher for a given α. This agrees with the normalized density $(\lambda_{sector}/\lambda_{detect})$ curve in Fig. 6, where the normalized density decreases with the average vacancy.

VIII. Conclusion

In this paper we have investigated the coverage problem for target localization applications. We have proposed two methods to derive the sensor density required for satisfactory localization in a field. In the first method we transform the problem into an equivalent disk coverage problem and show that we require 4 times more sensors in localization applications than that in detection applications. However, if we use the method of sector coverage, the density required is only 1.8 times more than that for detection. This shows that the conventional disk coverage model is insufficient for localization applications. Therefore, when we apply the disk model in coverage problems, we need to be aware of the trade-offs between the simple implementation of the disk model and the potential overestimation of the sensor density due to using this simplified model.

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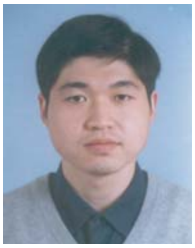

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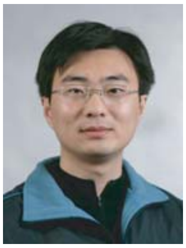

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Dr. Chua has carried out research in various areas of communication networks and has published more than 180 papers in these areas in international refereed journals and conferences. His current research interests are in wireless networks (in particular wireless sensor networks) and optical burst switched networks. He has also been an active member of the Institute of Electrical & Electronics Engineers (IEEE), Inc., and is a recipient of an IEEE 3rd Millennium medal.