Resource Allocation for Heterogeneous Multiuser OFDM-based Cognitive Radio Networks with Imperfect Spectrum Sensing

Shaowei Wang & Zhi-Hua Zhou*
National Key Laboratory
for Novel Software Technology
Nanjing University, Nanjing, China
Email: {wangsw, zhouzh}@lamda.nju.edu.cn

Mengyao Ge
School of Electronic
Science and Engineering
Nanjing University, Nanjing, China
Email: mengyao89@yahoo.cn

Chonggang Wang
InterDigital Communications
781 Third Avenue, King of Prussia
PA 19406, USA
Email: cgwang@ieee.org

***Abstract*—In this paper we study the resource allocation in OFDM-based cognitive radio (CR) networks, under the consideration of many practical limitations such as imperfect spectrum sensing, limited transmission power, different traffic demands of secondary users, etc. We formulated this general problem as a mixed integer programming task. Considering that this optimization task is computationally intractable, we propose to address it in two steps. For the first step, we perform subchannel allocation to satisfy heterogeneous users' rate requirement roughly and remove the integer constraints of the optimization problem. For the second step, we perform power allocation among the subchannels. By exploiting the problem structure to speedup the Newton step, we propose a Barrier-based method which is able to achieve the optimal power distribution with a complexity of $O(N)$, where N is the number of active OFDM subchannels, significantly better than the complexity of $O(N^3)$ of standard techniques. Moreover, we proposed a method which is able to approximate the optimal solution with a constant complexity. Numerical results validate that our proposal exploits the overall capacity of CR systems well subjected to different traffic demands of users.**

I. Introduction

With the ever increasing wireless applications, radio spectrum becomes more and more crowded, especially in the band below 6GHz. It has been disclosed by many studies that, however, large portions of spectrum are highly underutilized due to inefficient conventional regulatory policies [1]. Cognitive radio (CR) is deemed as a promising paradigm with great potential to improve the utilization of spectrum [2]. In a CR system, secondary users (SUs) are allowed to sense the spectrum registered by a licensed system and use the idle band of spectrum in an opportunistic spectrum access manner; that is, if an SU detects the presence of a licensed user (LU) in a given channel, it releases the channel and switches to a vacant channel, or waits in a pool if no vacant channel is available. However, owing to the inherent feedback delays, estimation errors and quantization errors in practical wireless systems, there are inevitable sensing errors, leading to heavy interference to the LU. To avoid unacceptable performance degradation of the LU, the interference generated by the SUs must be regularly controlled, and the physical layer of CR systems should be very flexible to meet these requirements.

Orthogonal Frequency Division Multiplexing (OFDM), which offers a high flexibility in radio resource allocation (RA), is widely recognized as a promising air interface of a CR system [3]. As one of the most important issues in OFDM systems, adaptive RA has been studied intensively during the past decade [4–8] and a comprehensive survey can be found in [9]. For the arising OFDM-based CR networks, dynamic RA is very important because it is the prerequisite to achieve high system performance, such as capacity and quality of service (QoS). RA in an OFDM-based CR network, however, is more complex than that in a conventional OFDM system because the LU may not adopt OFDM modulation, leading to the interference between the two systems. Moreover, the unavoidable sensing errors in the CR network can aggravate the interference, and the interference introduced to the LU must be carefully controlled below a threshold to prevent the degeneration of the performances of the LU's. Thus, many existing RA algorithms are no longer suitable for OFDM-based CR networks.

Adaptive RA for OFDM-based CR networks has attracted significant attention during recent years [10–21]. [22] provides a comprehensibly survey, and RA for single SU case is investigated in [10–13]. In [10], optimal and suboptimal power allocation schemes are studied, in which the sum capacity of a CR system is maximized under the interference constraint of the LU. In [11], a greedy heuristic algorithm, namely Max-Min, is proposed, which can produce solution close to the optimal. The computational load would be quite high if the algorithm is generalized to multiuser setting. In [12], an efficient algorithm is developed by jointly considering transmission power budget and interference constraints. The algorithm achieves a good tradeoff between sum capacity and complexity. In [13], a power allocation algorithm is derived, which can work out the optimal solution with low complexity.

*Corresponding Author. This work was partially supported by NSFC (61073097, 61021062), JiangsuSF (BK2011051, BK2011566), and Fundamental Research Funds for the Central Universities of China (1095021029, 1118021011).

RA algorithms for multiuser CR systems have been proposed in [14–16]. Interference issue is discussed in [14]. In [15], RA in a CR system with homogeneous non-real-time services is studied. In [16], a low complexity algorithm is proposed to maximize the sum capacity of a CR system while satisfying SUs' proportional rate requirements.

It is noteworthy that the spectrum sensing error has not been considered in [10–16]. In fact, perfect spectrum sensing is too difficult to achieve in practical wireless scenarios, and thus, RA with imperfect spectrum sensing should be considered. In [17], an optimal spectrum sharing policy is proposed to maximize the system throughput under alternatively perfect and imperfect sensing conditions. In [18], some metrics are derived to analyze the performance of a CR system. The results show that opportunistic spectrum access mode can significantly improve spectrum efficiency and system capacity, even under unreliable spectrum detection. In [19], an RA algorithm is proposed for a multiuser CR network under imperfect spectrum sensing, where the average system delay is optimized by allocating channels to adequate SUs through bipartite graph matching. In [20], both downlink and uplink RA algorithms are developed for an OFDM-based CR network operating in an opportunistic spectrum access manner, where the interference of out-of-band emissions and the spectrum sensing errors are considered. In [21], the vacant probabilities of subchannels are investigated, where the objective is to maximize the overall utility of the CR network while keeping the LU away from unacceptable interference.

In this paper, we take the mutual interference and the spectrum sensing errors into consideration in our system model. To support diverse services, we also model a heterogeneous CR network which serves for both real-time (RT) SUs and non-real-time (NRT) SUs. We try to maximize the sum rate of all SUs while guaranteeing the required rates of the RT users and a set of proportional rate constraints among the NRT users. These considerations lead to a general formulation of a mixed integer programming problem, which is computationally intractable. To make it tractable, we propose to address this problem in two steps. For the first step, we try to allocate subchannels based on channel gains and the interferences to LUs. For the second step, we try to allocate power to the subchannels. By exploiting the structure of the problem, we propose a method which is able to achieve the optimal power allocation, much more efficient than standard techniques. Furthermore, we develop a method that is able to achieve nearly optimal solution with a constant complexity. Numerical results validate the effectiveness and efficiency of our proposal.

The rest of this paper is organized as follows. In Section II, we illustrate the system model and formulate the problem as an optimization task. In Section III, we propose the subchannel allocation scheme. In Section IV, we develop an efficient *barrier* method for optimal power allocation. In Section V, we derive a suboptimal power allocation algorithm with lower complexity. Simulation results and discussions are given in Section VI, as well as discussions. Finally, we conclude the paper in Section VII.

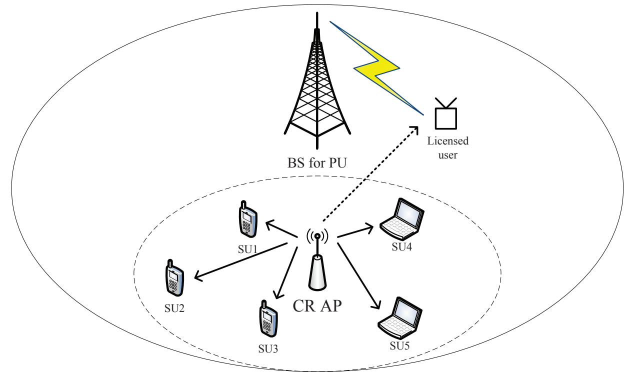

Fig. 1. System Model

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a CR system with K SUs denoted by $\mathcal{K} = \{1, 2, \ldots, K\}$. The SUs opportunistically use the spectrum of the LU via an access point (AP), as illustrated in Fig.1. Any part of the spectrum, named as a sub-band, can be used by the LU at any time. The bandwidth is divided into multiple OFDM subchannels in the CR network. With periodic spectrum sensing, the CR network identifies vacant sub-bands and selects a subset $\mathcal{N} = \{1, \ldots, N\}$ among the subchannels to transmit information. In other words, only the subchannels in vacant sub-bands can be used by the CR network. There are K_0 NRT users with proportional rate constraints, and $K - K_0$ RT users with fixed rate requirements R_k^{req}s. The bandwidth of each subchannel is B and the nominal spectrum of the nth subchannel spans from $f_s + (n-1)B$ to $f_s + nB$. When the CR system transmits information over the nth subchannel with unit transmission power, the interference introduced to the LU over the sub-band of the jth subchannel is [23]

$$I_j^n = \int_{(j-1)B-(n-1/2)B}^{jB-(n-1/2)B} g_n \phi(f) df, \tag{1}$$

where g_n is the power gain from the AP to the receivers of the LU on the nth subchannel. $\phi(f) = T(\frac{\sin \pi f T}{\pi f T})^2$ is the power spectrum density (PSD) of OFDM signal, T is OFDM symbol duration.

In practical systems, there are typically two kinds of sensing errors [18]. The first is *misdetection*, which occurs when the CR system fails to detect the LU signals. The sub-band of a subchannel is identified to be vacant but it is actually used by the LU. The other kind of sensing errors is *false alarm*, which means the CR network identifies a sub-band is unavailable but it is vacant actually. Generally, the AP in the CR system collects the sensed information of all SUs and makes a decision on which subchannel can be used by SUs. Then the set of available subchannels $\mathcal{N}$ is predicted, as well as the set of unavailable subchannels $\mathcal{N}_o$. The probabilities of misdetection and false alarm on the nth subchannel are Q_n^m and Q_n^f, respectively. Obviously, misdetection results in co-

TABLE I
PROBABILITY INFORMATION FROM IMPERFECT SPECTRUM SENSING

	Actual state	Sensing result	Probability information
1	Active ($\mathcal{H}_{1,n}$)	Occupied ($\mathcal{O}_{1,n}$)	$P\{\mathcal{O}_{1,n}\|\mathcal{H}_{1,n}\} = 1 - Q_n^m$
2	Active ($\mathcal{H}_{1,n}$)	Vacant ($\mathcal{O}_{2,n}$)	$P\{\mathcal{O}_{2,n}\|\mathcal{H}_{1,n}\} = Q_n^m$
3	Idle ($\mathcal{H}_{2,n}$)	Vacant ($\mathcal{O}_{2,n}$)	$P\{\mathcal{O}_{2,n}\|\mathcal{H}_{2,n}\} = 1 - Q_n^f$
4	Idle ($\mathcal{H}_{2,n}$)	Occupied ($\mathcal{O}_{1,n}$)	$P\{\mathcal{O}_{1,n}\|\mathcal{H}_{2,n}\} = Q_n^f$

channel interference to the LU, while false alarm lowers the utilization efficiency of the spectrum.

There are four possible scenarios for spectrum sensing, as summarized in the Table I, where $\mathcal{H}_{1,n}$ and $\mathcal{H}_{2,n}$ are the hypotheses of the presence and absence of the LU's signal on the nth subchannel, $\mathcal{O}_{1,n}$ and $\mathcal{O}_{2,n}$ are the events that the nth subchannel is unavailable or available based on the sensed infomation, respectively. Denote P_n^1 as the probability that the nth subchannel is truly used by the LU while the CR network makes a correct judgement and thus we have

$$\begin{aligned} P_n^1 &= P\{\mathcal{H}_{1,n}|\mathcal{O}_{1,n}\} \\ &= \frac{P\{\mathcal{O}_{1,n}|\mathcal{H}_{1,n}\}P\{\mathcal{H}_{1,n}\}}{P\{\mathcal{O}_{1,n}|\mathcal{H}_{1,n}\}P\{\mathcal{H}_{1,n}\} + P\{\mathcal{O}_{1,n}|\mathcal{H}_{2,n}\}P\{\mathcal{H}_{2,n}\}} \\ &= \frac{(1-Q_n^m)Q_n^L}{(1-Q_n^m)Q_n^L + Q_n^f(1-Q_n^L)}, \end{aligned} \tag{2}$$

where Q_n^L is the priori probability that the sub-band of the nth subchannel is used by the LU. Similarly, we can define P_j^2 as the probability that the jth subchannel is indeed occupied while the CR system deems it as vacant.

The interference introduced to the LU by the access of an SU on subchannel $n \in \mathcal{N}$ with unit transmission power is

$$I_n = \sum_{j\in\mathcal{N}_o} P_j^1 I_j^n + \sum_{j\in\mathcal{N}} P_j^2 I_j^n. \tag{3}$$

The transmission rate on subchannel n used by SU k is

$$r_{k,n} = \log\left(1 + \frac{p_{k,n}|c_{k,n}|^2}{\Gamma(BN_0 + I)}\right) \tag{4}$$

where $p_{k,n}$ is the power allocated to subchannel n, $c_{k,n}$ is the channel gain between the AP and the receiver of SU k over subchannel n, N_0 is the PSD of additive white Gaussian noise and Γ is the signal-to-noise ratio (SNR) gap. For an uncoded MQAM, Γ is related to a given bit-error-rate (BER) and $\Gamma = -\ln(5\text{BER})/1.5$ [24]. The interference caused by the LU signals is I, which can be regarded as noise and measured by the receivers of SUs.

Denote $H_{k,n} = \frac{|c_{k,n}|^2}{\Gamma(BN_0+I)}$. The rate of user k can be expressed as

$$R_k = \sum_{n=1}^{N} \rho_{k,n} \log(1 + p_{k,n}H_{k,n}), \tag{5}$$

where $\rho_{k,n}$ can either be 1 or 0 informing whether the subchannel n is occupied by the SU k or not. We try to maximize the downlink sum capacity of the AP while guaranteeing the rate requirements of all SUs, under power limitation and interference constraints, and the optimization problem is following,

$$\begin{aligned} \max_{\rho_{k,n},p_{k,n}} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{k,n}\log(1+p_{k,n}H_{k,n}) \\ s.t.\ C1 \quad & p_{k,n} \ge 0, \forall n \in \mathcal{N}, \forall k, \\ C2 \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{k,n}p_{k,n} \le P_T, \\ C3 \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{k,n}p_{k,n}I_n \le I_{th}, \\ C4 \quad & R_1 : R_2 : \ldots : R_{K_0} = \gamma_1 : \gamma_2 : \ldots : \gamma_{K_0}, \\ C5 \quad & R_k = R_k^{req}, k = K_0+1, \ldots, K, \\ C6 \quad & \rho_{k,n} \in \{0,1\}, \forall n \in \mathcal{N}, \forall k, \\ C7 \quad & \sum_{k=1}^{K} \rho_{k,n} = 1, \forall n \in \mathcal{N}, \end{aligned} \tag{6}$$

where P_T is the transmission power limit of the AP. C3 means that the interference to the LU can not exceed I_{th}. C4 is the proportional rate constraints of the NRT users. C5 is the fixed rate requirements of the RT users. C6 and C7 indicate that the subchannels are not shared among SUs.

III. INTEGER SUBCHANNEL ALLOCATION

Note that (6) defines a mixed integer programming problem that involves both binary variables $\rho_{k,n}$'s and real variables $p_{k,n}$'s for optimization. It is generally intractable. Furthermore, the objective function in (6) is nonconvex for $\{\rho_{k,n}, p_{k,n}\}$, hence, simply relaxing the integer constraints of $\rho_{k,n}$'s as real still suffers from nonconvex problem, which makes it more challenging to solve. One possible approach in dealing with nonconvex mixed integer programming problems is to apply the minimax convex relaxation technique [25]. However, it still needs to solve a series of convex optimization problems and it is not very efficient when the number of variables is as the case of our concerned problem.

To reduce complexity and make the problem trackable, we consider a two-stage approach where similar ideas have achieved success in diverse scenarios [26, 27]. Specifically, we separate the RA into two individual procedures, that is, subchannel allocation and power distribution. In this section, we first propose an efficient subchannel allocation scheme, which removes the integer constraints in (6) while roughly satisfying the rate constraints of the SUs.

In an OFDM-based CR network, the subchannel with the higher SNR may generate more interference to the LU if the sub-band of this subchannel is adjacent to the band used by the LU; it limits the maximum possible power over this subchannel. Hence, any integer-tone subchannel assignment procedure should consider the SNR and the interference to the LU. In other words, the possible maximum rate of a

TABLE II
SUBCHANNEL ALLOCATION

Algorithm 1
Initialization:
Current rate for each SU: $\mathcal{R} = \{R_1, \ldots, R_K\} = \mathbf{0}$
$\mathcal{N}_t = \mathcal{N}$, $\Omega_k = \emptyset, \forall k$
First round for RT users
While $\mathcal{N}_t \neq \emptyset$ and $\min(R_k - R_k^{req}) < 0$
Find k^* satisfies $R_{k^*}/R_{k^*}^{req} \leq R_k/R_k^{req}$, for RT users
For k^*, find n^* satisfies $r_{k^*,n^*}^{max} \geq r_{k^*,n}^{max}, \forall n \in \mathcal{N}_t$
$\mathcal{N}_t = \mathcal{N}_t/n^*$, $\Omega_{k^*} = \Omega_{k^*} \cup n^*$;
$R_{k^*} = R_{k^*} + \log(1 + p_{k^*,n^*} H_{k^*,n^*})$;
Endwhile
Second round for NRT users
While $\mathcal{N}_t \neq \emptyset$;
Find k^* satisfies $R_{k^*}/\gamma_{k^*} \leq R_k/\gamma_k$;
For k^*, find n^* satisfies $r_{k^*,n^*}^{max} \geq r_{k^*,n}^{max}, \forall n \in \mathcal{N}_t$;
$\mathcal{N}_t = \mathcal{N}_t/n^*$, $\Omega_{k^*} = \Omega_{k^*} \cup n^*$;
$R_{k^*} = R_{k^*} + r_{k^*,n^*}^{max}$;
Endwhile

subchannel is bounded by the SNR and the interference level,

$$r_{k,n}^M = \log(1 + p_{k,n}^M H_{k,n}). \quad (7)$$

$r_{k,n}^M$ can be regarded as the highest achievable rate of subchannel n used by SU k, where $p_{k,n}^M$ is the maximum possible power allocated to subchannel n,

$$p_{k,n}^M = \min(P_T, I_{th}/I_n). \quad (8)$$

The SNR of a subchannel and the interference to the LU are linked to a unified capacity in this way.

Since there are RT users and NRT users in the CR network, we separate the subchannel allocation procedure in two steps. First, the RT users have the priority to select subchannels until the rate requirements of all RT users are satisfied. Then the remaining subchannels are allocated to the NRT users based on their proportional rate constraints. The outline of our proposed subchannel allocation scheme is describe in Table II, where Ω_k denotes the set of subchannels allocated to user k.

At the first round, the RT user whose rate is the farthest away from its target rate has the priority to get a new subchannel among the available ones. Preferably, the subchannel with the highest achievable rate associated with this user will be chosen and the power distribution is temporarily set as $p_{k,n} = \min(P_T/N, I_{th}/I_n)$. Then the NRT user who suffers the severest unjustness, is given a privilege to choose a subchannel with the highest achievable rate from the remaining ones. The procedure terminates until all subchannels are consumed. Either the fixed rate requirements or the proportional rate constraints are roughly satisfied with such a subchannels allocation scheme. The exact satisfaction of the rate constraints will be ultimately accomplished after power disribution among subchannels.

IV. EFFICIENT BARRIER METHOD FOR OPTIMAL POWER ALLOCATION

Given a subchannel assignment, the binary variables $\rho_{k,n}$'s in (6) are fixed to 0 or 1, the integer constraints in (6) are removed and the remaining RA is the power distribution among subchannels. Recall that Ω_k is the set of subchannels allocated to SU k, the power distribution problem can be written as:

$$\begin{aligned} \max_{r_{k,n}} \quad & \sum_{k=1}^{K} \sum_{n \in \Omega_k} r_{k,n}, \\ s.t. \; C1 \quad & p_{k,n} \geq 0, \forall n \in \mathcal{N}, \forall k, \\ C2 \quad & \sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} \leq P_T, \\ C3 \quad & \sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} I_n \leq I_{th}, \\ C4 \quad & \sum_{n \in \Omega_k} r_{k,n} = \beta_k \sum_{n \in \Omega_1} r_{1,n}, k = 2, ..., K_0, \\ C5 \quad & \sum_{n \in \Omega_k} r_{k,n} = R_k^{req}, k = K_0 + 1, \ldots, K, \end{aligned} \quad (9)$$

where $\beta_k = \gamma_k/\gamma_1, k = 1, \ldots, K_0$. It is easy to prove that (9) defines a convex problem that can be solved by standard convex optimization techniques, such as *barrier* method [28].

A. The barrier method

Barrier method is a convex optimization algorithm. By introducing a logarithmic barrier function with parameter t, the barrier method makes the inequality constraints implicit in the optimization objective and converts the original problem into a sequence of linear equality constrained minimization problems, the solutions to which are called central points in the central path related to the original problem. The central point will be more accurately approximated to the optimal solution as the parameter t increases. For searching the central point with a given t, Newton method is generally employed. The computational complexity of the barrier method mainly lies in the computation of Newton step that needs matrix inversion with complexity of $O(N^3)$.

First, we convert all inequality constraints into a logarithmic barrier function $\phi(\boldsymbol{r})$,

$$\begin{aligned} \phi(\boldsymbol{r}) = \; & -\sum_{k=1}^{K} \sum_{n \in \Omega_k} \log r_{k,n} - \log(P_T - \sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n}) \\ & -\log(I_{th} - \sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} I_n), \end{aligned} \quad (10)$$

where all variables $r_{k,n}$'s are put into a unified vector $\boldsymbol{r}$, and $\boldsymbol{r} = \{r_n\}_{n=1}^N$. Notice that the subscript k can be omitted as it is fixed for a given subchannel assignment. Denote $f(\boldsymbol{r}) = \sum_{k=1}^{K} \sum_{n \in \Omega_k} r_{k,n}$. The minimization problem with a certain parameter t is

$$\begin{aligned} \min \quad & \psi_t(\boldsymbol{r}) = -tf(\boldsymbol{r}) + \phi(\boldsymbol{r}) \\ s.t. \quad & A\boldsymbol{r} = 0, \end{aligned} \quad (11)$$

where

$$A_{k,n} = \begin{cases} \beta_{k+1} & k = 1, \ldots, K_0 - 1, n \in \Omega_1 \\ -1 & k = 2, \ldots, K - 1, n \in \Omega_k \\ 0 & otherwise. \end{cases} \quad (12)$$

TABLE III
THE BARRIER METHOD

Algorithm 2

Initialization for the Barrier method
  Find strictly feasible point $\boldsymbol{r}$, $t := t^{(0)} > 0$, tolerance $\epsilon > 0, \mu > 1$
Outer Loop for Barrier method
  Centering step: Compute $\boldsymbol{r}^*(t)$
  Initialization for Newton method
    Starting point $\boldsymbol{r}$, subject to $A\boldsymbol{r} = 0$,
    tolerance $\epsilon_n > 0, \alpha \in (0, 1/2), \beta \in (0, 1)$
  Inner Loop for Newton method
    Compute $\Delta\boldsymbol{r}$ and $\lambda := - \bigtriangledown \psi_t(\boldsymbol{r})\Delta\boldsymbol{r}$;
    Quit if $\lambda^2/2 \leq \epsilon_n$
    Backtracking line search on $\psi_t(\boldsymbol{r})$, $s := 1$
      while $\psi_t(\boldsymbol{r} + s\Delta\boldsymbol{r}) > \psi_t(\boldsymbol{r}) - \alpha s \lambda^2$
        $s := \beta s$
      endwhile
    Update:$\boldsymbol{r} := \boldsymbol{r} + s\Delta\boldsymbol{r}$
  Update:$\boldsymbol{r}^*(t) = \boldsymbol{r}$.
  Stopping criterion:$(N + 2)/t < \epsilon$.
  Increase:$t := \mu t$.

The optimal solution to (11) is an approximation of the original problem. As t increases, the approximation is more and more closer to the optimal solution.

At the centering step of the barrier method, Newton method is employed to compute the central point. With a given parameter t, Newton step $\Delta\boldsymbol{r}$ and the associated dual variables ν are given by the following Karush-Kuhn-Tucker (KKT) systems,

$$\begin{bmatrix} \bigtriangledown^2\psi_t(\boldsymbol{r}) & A^T \\ A & \mathbf{0}_n \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{r} \\ \nu \end{bmatrix} = \begin{bmatrix} - \bigtriangledown \psi_t(\boldsymbol{r}) \\ \mathbf{0}_v \end{bmatrix}, \quad (13)$$

where $\mathbf{0}_n \in \mathcal{R}^{(K-1)\times(K-1)}$ and vector $\mathbf{0}_v \in \mathcal{R}^{(K-1)\times 1}$. $\bigtriangledown^2\psi_t(\boldsymbol{r})$ and $\bigtriangledown\psi_t(\boldsymbol{r})$ are the Hessian and the gradient of $\psi_t(\boldsymbol{r})$, respectively.

The outline of the Barrier method is summarized in Table III. ϵ and ϵ_n are the tolerances of the barrier method and the Newton step, respectively. α and β are two constants utilized in backtracking line search with $\alpha \in (0, 0.5)$ and $\beta \in (0, 1)$. The step size of the backtracking line search is s with $s > 0$. t and μ are parameters that are associated with a tradeoff between outer iterations and inner iterations.

B. Speedup the Newton step

Solving (13) directly has a complexity of $O((N + K)^3)$, which is too high to apply in practical CR networks. Indeed, the Newton step can be speedup by exploiting the structure of the problem (9). Denote

$$\begin{aligned} f_0 &= P_T - \sum_{k=1}^{K} \sum_{n\in\Omega_k} p_{k,n}, \\ f_l &= I_{th} - \sum_{k=1}^{K} \sum_{n\in\Omega_k} p_{k,n} I_n, \end{aligned} \quad (14)$$

the gradient and the Hessian of $\psi_t(\boldsymbol{r})$ are respectively given by

$$\bigtriangledown\psi_t(\boldsymbol{r}) = -t - \frac{1}{r_n} + \frac{e^{r_n}}{f_0 H_n} + \frac{I_n e^{r_n}}{f_l H_n}, \quad (15)$$

$$\begin{aligned} \bigtriangledown^2\psi_t(\boldsymbol{r}) &= \begin{bmatrix} 1/r_1^2 & & \\ & \ddots & \\ & & 1/r_N^2 \end{bmatrix} + \frac{\bigtriangledown^2 f_0}{f_0} \\ &\quad + \frac{\bigtriangledown f_0 \bigtriangledown f_0^T}{f_0^2} + \frac{\bigtriangledown f_l \bigtriangledown f_l^T}{f_l^2} + \frac{\bigtriangledown^2 f_l}{f_l} \quad (16) \\ &= \begin{bmatrix} D_1 & & \\ & \ddots & \\ & & D_N \end{bmatrix} + g_0 g_0^T + g_l g_l^T. \end{aligned}$$

The Hessian is positive definite because the diagonal matrix $D = diag(D_1, \ldots, D_N)$, $g_0 g_0^T$ and $g_l g_l^T$ are all positive definite matric. Moreover, since A is a full row rank matrix, the KKT matrix at the left-side of (13) is invertible.

Theorem *1:* The equation (13) can be solved with the complexity of $O(N)$.

The proof is presented in Appendix. The computational cost is significantly reduced by this algorithm compared to matrix inversion with complexity $O((N + K)^3)$.

V. A SUBOPTIMAL POWER ALLOCATION ALGORITHM

In this section, we propose a constant complexity power allocation algorithm to approach the optimal solution. Inspired by the index function concept proposed in [12], we introduce a normalized cost function to measure the cost of allocating a given rate over each subchannel,

$$F_c(r_{k,n}) = \frac{e^{r_{k,n}} - 1}{e^{r_{k,n}^M} - 1}. \quad (17)$$

The SNR of a subchannel and the interference generated to the LU are jointly considered in the normalized cost function. We can convert the power allocation problem into the following form:

$$\begin{aligned} \max_{r_{k,n}} \quad & \sum_{k=1}^{K} \sum_{n\in\Omega_k} r_{k,n} \\ s.t. \; C1 \quad & r_{k,n} \geq 0, \forall k, n \in \Omega_k \\ C2 \quad & \sum_{k=1}^{K} \sum_{n\in\Omega_k} F_c(r_{k,n}) \leq C \\ C3 \quad & \sum_{n\in\Omega_k} r_n = \beta_k \sum_{n\in\Omega_1} r_n, k = 2, ..., K_0 \\ C4 \quad & \sum_{n\in\Omega_k} r_n = R_k^{req}, k = K_0 + 1, \ldots, K, \end{aligned} \quad (18)$$

where C in C2 is a constant jointly determined by the transmission power and the interference constraints, informing the maximum sum capacity of the CR system. The solution of (18) can be regarded as an approximation to (9). Notice that we do not know the exact value of C in advance. It can be worked out when all $r_{k,n}$'s are obtained. We will show that it is not necessary to know C when solving (18).

The Lagrangian of (18) is

$$\begin{aligned} L = & -\sum_{k=1}^{K}\sum_{n\in\Omega_k} r_{k,n} + \lambda(\sum_{k=1}^{K}\sum_{n\in\Omega_k} F_c(r_{k,n}) - C) \\ & + \sum_{k=2}^{K_0}\mu_k(\beta_k\sum_{n\in\Omega_k} r_{k,n} - \sum_{n\in\Omega_k} r_{k,n}) \\ & + \sum_{k=K_0+1}^{K}\mu_k(R_k^{req} - \sum_{n\in\Omega_k} r_{k,n}), \end{aligned} \tag{19}$$

where $\lambda > 0$ and $\{\mu_k\}_{k=2}^{K}$ are the Lagrange multipliers. The KKT conditions of (19) are

$$\begin{aligned} \frac{\partial L}{\partial r_{1,n}} &= -1 + \lambda\frac{e^{r_{1,n}}}{e^{r_{1,n}^M}-1} + \sum_{k=2}^{K_0}\mu_k\beta_k = 0, n\in\Omega_1 \\ \frac{\partial L}{\partial r_{k,n}} &= -1 + \lambda\frac{e^{r_{k,n}}}{e^{r_{k,n}^M}-1} - \mu_k = 0, k\geq 2, n\in\Omega_k. \end{aligned} \tag{20}$$

Redefine $r_{k,n}$ as the rate on the nth subchannel used by the kth SU, the rate distribution for each SU satisfies

$$r_{k,n} - r_{k,m} = \log(\frac{p_{k,n}^M H_{k,n}}{p_{k,m}^M H_{k,m}}), k = 1,\ldots,K. \tag{21}$$

Without loss of generality, assuming $r_{k,1}^M \leq r_{k,2}^M \leq \ldots \leq r_{k,N_k}^M$ and denoting $v_{k,n} = \frac{p_{k,n}^M H_{k,n}}{p_{k,1}^M H_{k,1}}$, we have

$$r_{k,n} = r_{k,1} + \log v_{k,n}, n = 1,\ldots,N_k, \tag{22}$$

where N_k is the number of subchannels allocated to the kth SU. Consequently, the rate of the kth usercan be calculated as

$$R_k = N_k r_{k,1} + \sum_{n=1}^{N_k}\log v_{k,n}, \tag{23}$$

and the power consumption and interference generated to the LU by each SU are given by

$$\begin{aligned} P_k &= \sum_{n=1}^{N_k} e^{r_{k,1}}\frac{v_{k,n}}{H_{k,n}} - \sum_{n=1}^{N_k}\frac{1}{H_{k,n}} = e^{r_{k,1}}X_k^0 - Y_k^0 \\ I_k &= \sum_{n=1}^{N_k} e^{r_{k,1}}\frac{v_{k,n}I_n}{H_{k,n}} - \sum_{n=1}^{N_k}\frac{I_n}{H_{k,n}} = e^{r_{k,1}}X_k^1 - Y_k^1. \end{aligned} \tag{24}$$

As fixed rate requirements are imposed to the RT users, we can obtain the rate allocation for the RT users from

$$N_k r_{k,1} + \sum_{n=1}^{N_k}\log v_{k,n} = R_k^{req}. \tag{25}$$

Then other $r_{k,n}$'s for the kth user can be easily derived from (21).

Denote $P_r = \sum_{k=K_0+1}^{K} P_k$ and $I_r = \sum_{k=K_0+1}^{K} I_k$ as the power consumed and the interference introduced to the LU by the RT users, respectively. For the NRT users, there is a set of proportional rate constraints to be satisfied. From the equation $\sum_{n=1}^{N_k} r_{k,n} = \beta_k\sum_{n=1}^{N_1} r_{1,n}$, we have

$$\begin{aligned} r_{k,1} &= \frac{N_1\beta_k}{N_k}r_{1,1} + \frac{1}{N_k}(\beta_k\sum_{n=1}^{N_1} v_{1,n} - \sum_{n=1}^{N_k} v_{k,n}) \\ &= V_k r_{1,1} + W_k. \end{aligned} \tag{26}$$

Consequently,

$$\begin{aligned} P_k &= e^{V_k r_{1,1}+W_k}X_k^0 - Y_k^0 \\ I_k &= e^{V_k r_{1,1}+W_k}X_k^1 - Y_k^1. \end{aligned} \tag{27}$$

According to the transmission power limit and the interference threshold, the rate $r_{1,1}$ is constrained by

$$\begin{aligned} \sum_{k=1}^{K_0} e^{V_k r_{1,1}+W_k}X_k^0 - Y_k^0 \leq P_T - P_r, \\ \sum_{k=1}^{K_0} e^{V_k r_{1,1}+W_k}X_k^1 - Y_k^1 \leq I_{th} - I_r. \end{aligned} \tag{28}$$

The maximum value of $r_{1,1}$ can be worked out by using bisection method; other $r_{k,n}$'s are obtained by inserting $r_{1,1}$ into (22).

We also need to consider the constraints that $r_{k,n} \geq 0$ for both the RT and the NRT users. For subchannel n, there is no power allocation if $r_{k,n} < 0$, which may occur when SU k does not have a high achievable rate on this subchannel. In this case, we should give up using this subchannel and update the rate allocation.

The computational complexity can be estimated roughly as follows. As the rate allocation for the RT users can be obtained from (22) and (25) directly, the computational cost mainly lies in the rate allocation for the NRT users. We need to solve the two inequalities in (28) by using bisection method. The number of iterations is about $2\log_2(1/\epsilon_b)$, where ϵ_b is the error tolerance for the bisection method. Hence the complexity of the proposed suboptimal power allocation algorithm is $O(\log_2(1/\epsilon_b))$, which is a constant for a given ϵ_b.

VI. Simulation results

Experiments are conducted to evaluate the performance of our proposed algorithms. Consider a multiuser OFDM-based CR system, where all users randomly locate in an area of 3×3km, and each SU's receiver uniformly is distributed in a circle within 0.5km from its transmitter. The path loss exponent is 4, the variance of shadowing effect is 10dB and the amplitude of multipath fading is Rayleigh. The noise power is 10^{-13}W and the interference threshold of the LU is 5×10^{-13}W. The activity probability (Q_n^L), misdetection (Q_n^m) and false alarm (Q_n^f) are uniform distributed over [0,1], [0.01,0.05] and [0.05,0.1], respectively. The AP in the CR network identifies available subchannels randomly.

Fig.2 shows the interference to the LU with and without considering sensing errors. There are 16 subchannels and the number of SUs is $K = 4$, including $K_0 = 2$ NRT users. The unit of x-axis is Watt. As depicted in Fig.2, once considering sensing errors, the interference to the LU is always kept below the threshold. On the other hand, if ignoring the sensing errors, the interference may exceed the threshold, which is mainly caused by the SU's access to the subchannels used by the LU. The LU may experience severely performance degradation when the co-channel interference occurs as a result of misdetection.

To evaluate sum capacity, we compare our proposed algorithms, including integer subchannel assignment proposed in

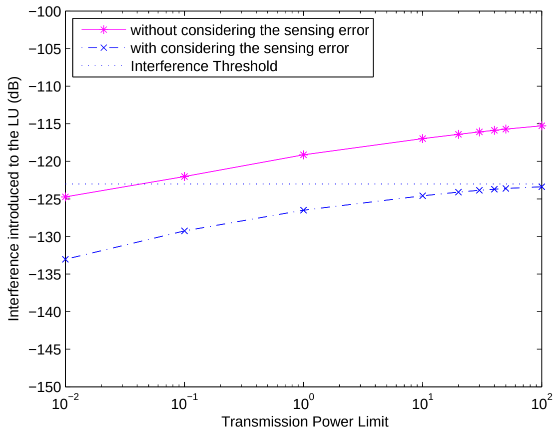

Fig. 2. Interference to the LU as a function of transmission power limit. ($K = 4$, $K_0 = 2$).

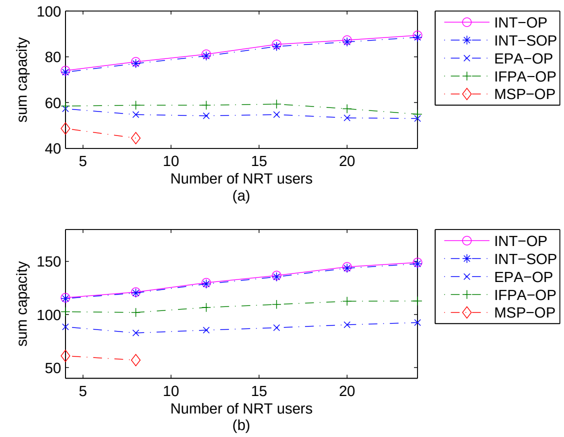

Fig. 4. Sum capacity as a function of the number of the NRT users. ($K - K_0 = 4$, $R_k^{req} = 5$bit/symbol, (a) $N = 64$; (b) $N = 128$).

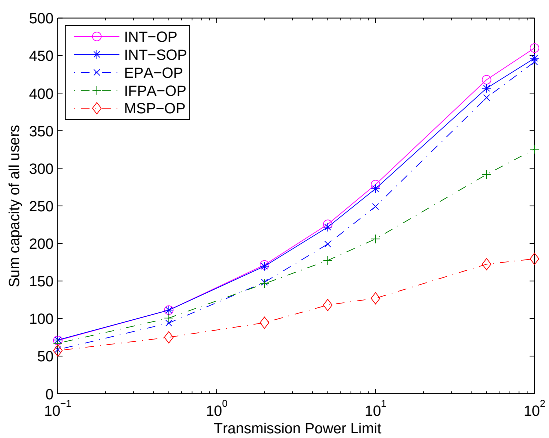

Fig. 3. Sum capacity as a function of transmission power limit. ($K = 4$, $K_0 = 2$).

section III with optimal power allocation proposed in IV (INT-OP), integer subchannel assignment with rate loading scheme proposed in V (INT-SOP), with the other three algorithms: EPA, IFPA and MSP. The EPA and the IFPA are introduced in [20]. The EPA assumes that equal power is distributed among subchannels, while IFPA allocates power inversely proportional to the interference level. The MSP always allocates a subchannels to the user who acquires the highest SNR over this channel. All schemes adopt optimal power allocation proposed in IV. There are 128 subchannels, $K = 4$, $K_0 = 2$, $\gamma_1 : \gamma_2 = 1 : 1$ and $R_{k,min} = 10$ bit/symbol. Fig.3 shows the sum capacity as a function of transmission power limit P_T. The unit of x-axis and y-axis are Watt and bit/symbol, respectively.

From Fig.3 we can observe that the sum capacity of the NRT users grows with the increase of power budget. Our proposed algorithms, the INT-OP and the INT-SOP perform better than the others. When power budget is small, the IFPA works quite well. The EPA obtains solutions close to our proposed schemes when P_T is large. This is because that the power limit and the interference level are jointly considered in our proposed subchannel allocation schemes, while the EPA and the IFPA take only one of them into consideration. It is worth noticing that the gap between the INT-OP and the INT-SOP is small, suggesting that the suboptimal power allocation algorithm proposed in Section V provides a good approximation to the optimal.

We also validate the effect of multiuser diversity for the CR network. Fig.4 shows the sum capacity of the CR network as a function of the number of the NRT users. There are 4 RT users with fixed rate requirement of $R_k^req = 5$bit/symbol. $P_T = 1W$ and $\gamma_1 : \gamma_2 : \ldots : \gamma_{K_0} = 1 : 1 : \ldots : 1$. The number of available subchannels is $N = 64$ and $N = 128$ in Fig.4(a) and Fig.4(b), respectively. The unit of y-axis is bit/symbol. From Fig.4 we can see that the sum capacity increases as K_0 grows for both the INT-OP and the INT-SOP. The CR network benefits from multiuser diversity because a subchannel is more likely to be allocated to an SU that has good channel gain over it. Furthermore, the INT-OP and the INT-SOP perform much better than the others as can be seen form Fig.4. The EPA-OP and the IFPA-OP could not obtain enough multiuser diversity, while the MSP-OP fails to adapt to the circumstance when the number of the NRT users is large because there are not enough available subchannels to maintain the proportional constraints.

The performance of the fixed rate requirements of the RT users and the proportional rate fairness among the NRT users are shown in Fig.5(a) and Fig.5(b), respectively. We compare our proposed two schemes with sum capacity maximization scheme, which maximizes the sum capacity without any rate constraints. $R_k^{req} = 40$bit/symbol, $\gamma_1 : \gamma_2 = 1 : 1$ and $\gamma_1 : \gamma_2 = 1 : 4$ in Fig.5(a) and Fig.5(b), respectively. $K = 4$ and $K_0 = 2$. The users with indices 3 and 4 in Fig.5 are the RT ones. The unit of y-axis is bit/symbol. It is shown that our proposed scheme can strictly satisfy the various rate requirements, which is difficult for the sum capacity maximization method.

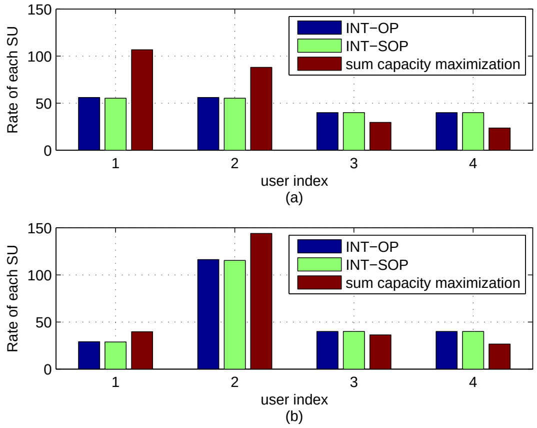

Fig. 5. Normalized rate distribution among SUs. ($K = 4$, $K_0 = 2$, $R_k^{req} =$ 40bit/symbol. (a) $\gamma_1 : \gamma_2 = 1 : 1$; (b) $\gamma_1 : \gamma_2 = 1 : 4$).

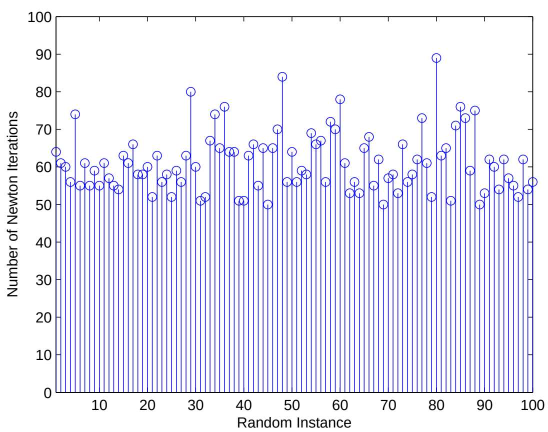

Fig. 6. Number of Newton iterations required for convergence during 100 channel realizations. ($K = 4$, $K_0 = 2$).

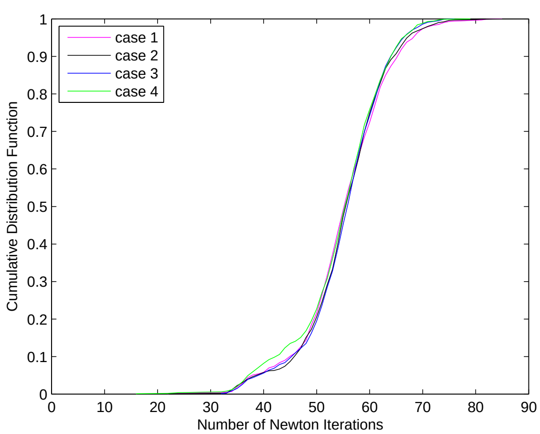

Fig. 7. CDF of the number of Newton iterations required for convergence over 1000 channel realizations with $K = 4$, $K_0 = 2$, $P_T =$ 1W. $\gamma_1 : \gamma_2 = 1 : 1$ (case 1), $\gamma_1 : \gamma_2 = 1 : 2$ (case 2), $\gamma_1 : \gamma_2 = 1 : 3$ (case 3), $\gamma_1 : \gamma_2 = 1 : 4$ (case 4)

Finally, we investigate the convergence of our proposed barrier method. As discussed in Section IV, the computational load of the barrier method mainly lies in the computation of Newton step. If the number of Newton iterations is large or varies in a wide range, the algorithm would be difficult to be applied in practical wireless systems. Fig.6 and Fig.7 show that it is not the case for our proposed barrier method. Fig.6 shows the number of Newton iteration for the barrier method to converge in 100 random instances. Fig.7 shows the cumulative distribution function (CDF) of the number of Newton iterations. The number of Newton iterations is less then 60 for more than 90% of instances and varies in a narrow range. Our proposed barrier method is effective and efficient.

VII. Conclusion

In this paper, we study the adaptive resource allocation problem in multiuser OFDM-based CR networks with heterogeneous services and imperfect spectrum sensing. Since we have considered many practical limitations, our general formulation leads to a challenging mixed integer programming problem which is computationally intractable. To make it tractable, we propose to address the problem in a two-step style. In the first step, we do subchannel allocation, by considering different requirements of real-time and non-real-time users. Then, in the second step, we do power allocation for subchannels. We propose a Barrier-based method which can achieve the optimal solution with a complexity of $O(N)$, much better than $O(N^3)$ of standard techniques, by exploiting problem struture to speedup the Newton step. Furthermore, we propose a method which can achieve nearly optimal solution with a constant complexity. Numerical simulations validate the effectiveness and efficiency of our proposed methods.

Acknowledgement

The authors are grateful to Yu-Feng Li and De-Chuan Zhan for helpful discussions.

Appendix

Proof of Theorem 1: Rewrite the KKT system (13) as the following form,

$$\Lambda_0 \tilde{x} = G_0, \tag{29}$$

where $\tilde{x} = \begin{bmatrix} \Delta \boldsymbol{r} \\ \nu \end{bmatrix}$ and $G_0 = \begin{bmatrix} -\bigtriangledown \Psi_t \\ \mathbf{0}_v \end{bmatrix}$. According to the decomposition in (16), Λ_0 can be written as

$$\Lambda_0 = \begin{bmatrix} D & A^T \\ A & \mathbf{0}_n \end{bmatrix} + G_1 G_1^T + G_2 G_2^T, \tag{30}$$

where $G_1 = \begin{bmatrix} g_0 \\ \mathbf{0}_v \end{bmatrix}$ and $G_2 = \begin{bmatrix} g_l \\ \mathbf{0}_v \end{bmatrix}$.

As discussed in Section IV, we can derive an efficient method the calculate the Newton step. A two-step decomposition is required as shown in the following,

Step 1 $\Lambda_0 = \Lambda_1 + G_1 G_1^T$,

where $\Lambda_1 = \begin{bmatrix} D & A^T \\ A & \mathbf{0}_n \end{bmatrix} + G_2 G_2^T$

Particularly we have $\tilde{x} = \upsilon_1^1 - \frac{G_1\upsilon_1^1}{1+G_1\upsilon_2^1}\upsilon_2^1$,
where $\Lambda_1\upsilon_1^1 = G_0$ and $\Lambda_1\upsilon_2^1 = G_1$.

Step 2 $\Lambda_1 = \Lambda_2 + G_2G_2^T$,
where $\Lambda_1 = \begin{bmatrix} D & A^T \\ A & \mathbf{0}_n \end{bmatrix}$
Similarly, $\upsilon_i^1 = \upsilon_i^2 - \frac{G_2\upsilon_i^2}{1+G_2\upsilon_3^2}\upsilon_3^2, i = 1, 2,$
and $\Lambda_2\upsilon_i^2 = G_{i-1}, i = 1, 2, 3.$

To calculate the Newton step, we need to solve the three equations listed in *Step 2*. Consider the equations in a unified form as follows,

$$\begin{bmatrix} D & A^T \\ A & \mathbf{0}_n \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} h \\ \mathbf{0}_v, \end{bmatrix}. \tag{31}$$

where $u \in \mathcal{R}^{M\times 1}$ and $v \in \mathcal{R}^{K-1\times 1}$. We have

$$\begin{aligned} & D_nu_n + \sum_{k=2}^{K_0}\beta_k v_{k-1} = h_n, n = 1, \ldots N_1 \\ & D_{\pi_k+n}u_{\pi_k+n} - v_{k-1} = h_{\pi_k+n}, k \geq 2, n = 1, \ldots, N_k \\ & \beta_k\sum_{n=1}^{N_1}u_n - \sum_{n=1}^{N_k}u_{\pi_k+n} = 0, k = 2, \ldots, K_0 \\ & \sum_{n=1}^{N_k}u_{\pi_k+n} = 0, k = K_0+1, \ldots, K, \end{aligned} \tag{32}$$

where $\pi_k = \sum_{i=1}^{k-1}N_i$. Denote $X_k = \sum_{n=1}^{N_k}u_{\pi_k+n}$, we have

$$\begin{aligned} X_1 &= \sum_{n=1}^{N_1}\frac{h_n - \sum_{k=2}^{K_0}\beta_k v_{k-1}}{H_n} = B_1 - A_1\sum_{k=2}^{K_0}\beta_k v_{k-1} \\ X_k &= \sum_{n=1}^{N_k}\frac{h_{\pi_k+n}+v_{k-1}}{H_{\pi_k+n}} = B_k + v_{k-1}A_k, k \geq 2, \end{aligned} \tag{33}$$

where $A_k = \sum_{n=1}^{N_k}\frac{1}{H_{\pi_k+n}}$ and $B_k = \sum_{n=1}^{N_k}\frac{h_{\pi_k+n}}{H_{\pi_k+n}}, k = 1, \ldots, K.$ Then we can get

$$v_{k-1} = \begin{cases} \frac{\beta_k X_1 - B_k}{A_k}, & k = 2, \ldots, K_0, \\ -\frac{B_k}{A_k} & k = K_0+1, \ldots, K. \end{cases} \tag{34}$$

Substituting the v_k's back into (33), we have

$$X_1 = (B_1 + A_1\sum_{k=2}^{K_0}B_k\beta_k/A_k)/(1 + A_1\sum_{k=2}^{K_0}\beta_k^2/A_k). \tag{35}$$

Then all variable v_k's can be worked out according to (34) and (35);other variables can also be obtained by substituting v_k's into (32).

References

[1] Federal Communications Commission, "Facilitating opportunities for flexible, efficient,and reliable spectrum use employing cognitive radio technologies," *FCC Report*, ET Docket 03-322, Dec. 2003.

[2] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. on Sel. Areas in Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[3] T. A. Weiss and F. K. Jondral, "Spectrum pooling: an innovative strategy for the enhancement of spectrum efficiency," *IEEE Commun. Mag.*, vol. 42, no. 3, pp. 8–14, Mar. 2004.

[4] C. Y. Wong, R. S. Cheng, K. B. Lataief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. on Sel. Areas in Commun.*, vol. 17, no. 10, pp. 1747–1758, Oct. 1999.

[5] J. Jang and K. B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE J. on Sel. Areas in Commun.*, vol. 21, no. 2, pp. 171–178, Feb. 2003.

[6] Z. Shen, J. G. Andrews, and B. L. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints," *IEEE Trans. on Wire. Commun.*, vol. 4, no. 6, pp. 2726–2737, Nov. 2005.

[7] M. Tao, Y.-C. Liang, and F. Zhang, "Resource allocation for delay differentiated traffic in multiuser OFDM systems," *IEEE Trans. on Wire. Commun.*, vol. 7, no. 6, pp. 2190–2201, June 2008.

[8] R. Madan, S. Boyd, and S. Lall, "Fast algorithms for resource allocation in wireless cellular networks," *IEEE/ACM Trans. on Networking*, vol. 18, no. 3, pp. 973–984, June 2010.

[9] S. Sadr, A. Anpalagan, and K. Raahemifar, "Radio resource allocation algorithms for the downlink of multiuser OFDM communication systems," *IEEE Commun. Surv. & Tutor.*, vol. 11, no. 3, pp. 92–106, Sep. 2009.

[10] G. Bansal, M. J. Hossain, and V. K. Bhargava, "Optimal and suboptimal power allocation schemes for OFDM-based cognitive radio systems," *IEEE Trans. on Wire. Commun.*, vol. 7, no. 11, pp. 4710–4718, Nov. 2008.

[11] Y. Zhang and C. Leung, "Resource allocation in an OFDM-based cognitive radio system," *IEEE Trans. on Commun.*, vol. 57, no. 7, pp. 1928–1931, July 2009.

[12] S. Wang, "Efficient resource allocation algorithm for cognitive OFDM systems," *IEEE Commun. Lett.*, vol. 14, no. 8, pp. 725–727, Aug. 2010.

[13] S. Wang, F. Huang, and Z.-H. Zhou, "Fast power allocation algorithm for cognitive radio networks," *IEEE Commun. Lett.*, vol. 15, no. 8, pp. 845–847, Aug. 2011.

[14] A. Attar, O. Holland, M. R. Nakhai, and A. H. Aghvami, "Interference-limited resource allocation for cognitive radio in orthogonal frequency-division multiplexing networks," *IET Commun.*, vol. 2, no. 6, pp. 806–814, July 2008.

[15] Y. Zhang and C. Leung, "Resource allocation for non-real-time services in OFDM-based cognitive radio systems," *IEEE Commun. Lett.*, vol. 13, no. 1, pp. 16–18, Jan. 2009.

[16] S. Wang, F. Huang, M. Yuan, and S. Du, "Resource allocation for multiuser cognitive OFDM networks with proportional rate constraints," *Int. J. of Commun. Sys.*, DOI:10.1002/dac.1272, 2011 (Online).

[17] S. Srinivasa and S. Jafar, "How much spectrum sharing is optimal in cognitive radio networks?" *IEEE Trans. on Wire. Commun.*, vol. 7, no. 10, pp. 4010–4018, Oct. 2008.

[18] S. Tang and B. Mark, "Modeling and analysis of opportunistic spectrum sharing with unreliable spectrum sensing," *IEEE Trans. on Wire. Commun.*, vol. 8, no. 4, pp. 1934–1943, Apr. 2009.

[19] L. Wu, W. Wang, H. Luo, G. Yu, and Z. Zhang, "Optimal resource allocation for cognitive radio networks with imperfect spectrum sensing," in *Proc. of the IEEE VTC 2010-Spring*, Taipei, China.

[20] S. M. Almalfouh and G. L. Stuber, "Interference-aware radio resource allocation in OFDMA-based cognitive radio networks," *IEEE Trans. on Veh. Tech.*, vol. 60, no. 4, pp. 1699–1713, May 2011.

[21] X. Zhou, G. Li, D. Li, D. Wang, and A. Soong, "Probabilistic resource allocation for opportunistic spectrum access," *IEEE Trans. on Wire. Commun.*, vol. 9, no. 9, pp. 2870–2879, Sep. 2010.

[22] R. Zhang, Y.-C. Liang, and S. Cui, "Dynamic resource allocation in cognitive radio networks," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 102–114, May 2010.

[23] G. Bansal, M. Hossain, and V. Bhargava, "Adaptive power loading for OFDM-based cognitive radio systems," in *Proc. of the IEEE ICC 2007*, June 2007, pp. 5137–5142.

[24] A. J. Goldsmith and S.-G. Chua, "Variable-rate variable-power MQAM for fading channels," *IEEE Trans. on Commun.*, vol. 45, no. 10, pp. 1218–1230, Oct. 1997.

[25] Y.-F. Li, I. W. Tsang, J. T. Kwok, and Z.-H. Zhou, "Tighter and convex maximum margin clustering," in *Proc. of the AISTATS 2009*, Clearwater Beach, FL, pp. 328–335.

[26] Z.-H. Zhou and Y. Jiang, "Nec4.5: Neural ensemble based c4.5," *IEEE Trans. on Know. and Data Eng.*, vol. 16, no. 6, pp. 770–773, June 2004.

[27] Y.-F. Li and Z.-H. Zhou, "Towards making unlabeled data never hurt," in *Proc. of the ICML 2011*, Bellevue, WA.

[28] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge,U.K., Cambridge University Press, 2004.