Resource Allocation for Heterogeneous Cognitive Radio Networks with Imperfect Spectrum Sensing

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***Abstract*—In this paper we study the Resource Allocation (RA) in Orthogonal Frequency Division Multiplexing (OFDM)-based Cognitive Radio (CR) networks, under the consideration of many practical limitations such as imperfect spectrum sensing, limited transmission power, different traffic demands of secondary users, etc. The general RA optimization framework leads to a complex mixed integer programming task which is computationally intractable. We propose to address this hard task in two steps. For the first step, we perform subchannel allocation to satisfy heterogeneous users' rate requirements roughly and remove the intractable integer constraints of the optimization problem. For the second step, we perform power distribution among the OFDM subchannels. By exploiting the problem structure to speedup the Newton step, we propose a barrier-based method which is able to achieve the optimal power distribution with an almost linear complexity, significantly better than the complexity of standard techniques. Moreover, we propose a method which is able to approximate the optimal solution with a constant complexity. Numerical results validate that our proposal exploits the overall capacity of CR systems well subjected to different traffic demands of users and interference constraints with given power budget.**

***Index Terms*—Barrier method, Cognitive Radio, Mixed integer programming, OFDM, and Resource Allocation.**

I. Introduction

With the ever increasing wireless applications, radio spectrum becomes more and more crowded, especially in the band below 6GHz. It has been disclosed by many studies that, however, large portions of spectrum are highly underutilized due to inefficient conventional regulatory policies [1]. Cognitive Radio (CR) is deemed as a promising paradigm with great potential to improve the utilization of spectrum [2]. In a CR system, Secondary Users (SUs) are allowed to sense the spectrum registered by Primary Users (PUs) and use the idle band of spectrum in an opportunistic spectrum access manner [3]; that is, if an SU detects the presence of a PU in a given channel, it releases the channel and switches to a vacant channel, or waits in a pool if no vacant channel is available. However, owing to the inherent feedback delays, estimation errors and quantization errors in practical wireless systems, there are inevitable sensing errors, leading to heavy interference to the PUs. To avoid unacceptable performance degradation of PUs, the interference generated by the SUs must be regularly controlled, and the physical layer of CR systems should be very flexible to meet these requirements.

Orthogonal Frequency Division Multiplexing (OFDM), which offers a high flexibility in radio Resource Allocation (RA), is widely recognized as a promising air interface of a CR system [4]. As one of the most important issues in OFDM systems, adaptive RA has been studied intensively during the past decade [5–9] and a general survey can be found in [10]. For the arising OFDM-based CR networks, dynamic RA is very important because it is the prerequisite to achieve high system performance, such as capacity and Quality of Service (QoS). RA in an OFDM-based CR network, however, is more complex than that in a conventional OFDM system because the PUs may not adopt OFDM modulation, leading to the interference between the two systems. Moreover, the unavoidable sensing errors in the CR network can aggravate the interference, and the interference introduced to the PUs must be carefully controlled below a threshold to prevent the degeneration of the performances of the PUs. Thus, many existing RA algorithms are no longer suitable for OFDM-based CR networks.

Adaptive RA in CR networks has attracted significant attention during recent years [11–25], and [26] provides a comprehensive survey. RA for single SU case in OFDM-based CR networks is investigated in [11–14]. In [11], optimal and suboptimal power allocation schemes are studied, in which the sum capacity of a CR system is maximized under the interference constraint of the PUs. In [12], a greedy heuristic algorithm, namely Max-Min, is proposed, which can produce solution close to the optimal. The computational load would be quite high if the algorithm is generalized to multiuser setting. In [13], an efficient algorithm is developed by jointly considering transmission power budget and interference constraints. The algorithm achieves a good tradeoff between sum capacity and complexity. In [14], a power allocation algorithm is derived, which can work out the optimal solution with low complexity. RA algorithms for multiuser CR systems have been proposed in [15–17]. In [15], RA in a CR system with homogeneous non-real-time services is studied. Both real-time and non-real-time services are considered in [16], and fast RA algorithms are developed. However, fairness among users is ignored in [16], as well as spectrum sensing issue. In [17], a low complexity algorithm is proposed to maximize the sum

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capacity of a CR system while satisfying SUs' proportional rate requirements. But there is a significant capacity gap between the proposed algorithm and the optimal. An optimal algorithm is developed in [18] to tackle the same problem. However, users' rate requirements are still not mentioned. In [19, 20], radio spectrum resource allocation for multi-hop CR networks is studied, in which near optimal algorithms are developed to address the formulated optimization problem.

It is noteworthy that the spectrum sensing error has not been considered in [11–20]. In fact, perfect spectrum sensing is too difficult to acquire in practical wireless scenarios, and thus, RA with imperfect spectrum sensing should be considered [27]. In [21], an optimal spectrum sharing policy is proposed to maximize the system throughput under alternatively perfect and imperfect sensing conditions. In [22], some metrics are derived to analyze the performance of a CR system. The results show that opportunistic spectrum access mode can significantly improve spectrum efficiency and system capacity, even under unreliable spectrum detection. In [23], an RA algorithm is proposed for a multiuser CR network under imperfect spectrum sensing, where the average system delay is optimized by allocating channels to adequate SUs through bipartite graph matching. In [24], both downlink and uplink RA algorithms are developed for an OFDM-based CR network operating in an opportunistic spectrum access manner, where the interference of out-of-band emissions and the spectrum sensing errors are considered. However, users' rate requirements are not mentioned in [24]. In [25], the vacant probabilities of subchannels are investigated, where the objective is to maximize the overall utility of the CR network while keeping the PUs away from unacceptable interference.

In this paper, the mutual interference and the spectrum sensing errors are taken into consideration in our system model. To support diverse services, we also model a heterogeneous CR network which serves for both Real-Time (RT) SUs and Non-Real-Time (NRT) SUs. We try to maximize the sum rate of all SUs while guaranteeing the required rates of the RT users and a set of proportional rate constraints among the NRT users to make the resource allocation much fairer. These considerations lead to a general formulation of a mixed integer programming problem, which is computationally intractable. To make it tractable, we propose to address this problem in two steps. For the first step, we try to allocate subchannels based on channel gains and the interferences to PUs. For the second step, we try to allocate power among the OFDM subchannels. By exploiting the structure of the problem, we propose a method which is able to achieve the optimal power allocation, much more efficient than standard techniques. Furthermore, we develop a method that is able to achieve nearly optimal solution with a constant complexity. Numerical results validate the effectiveness and efficiency of our proposed algorithms. Our system model is general and many other RA problems in the literature fall into the formulated optimization framework in this work. So our proposed algorithms are suitable for a wide range of RA scenarios in CR systems.

The rest of this paper is organized as follows. In Section II, we illustrate the system model and formulate the problem as an optimization task. In Section III, we propose the subchannel allocation scheme. In Section IV, we develop an efficient *barrier* method for the optimal power allocation. In Section V, we derive a suboptimal power allocation algorithm with lower complexity. Simulation results are given in Section VI, as well as discussions. Finally, we conclude the paper in Section VII.

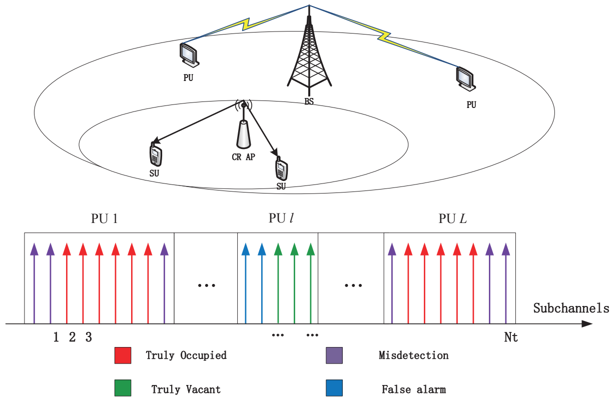

Fig. 1. System Model

II. System Model and Problem Formulation

To make the rest of this paper easy to follow, we list some frequently used symbols terminologies and symbol notations in Table I.

Consider two cellular systems, one of which is for PUs and the other is for K CR users denoted by $\mathcal{K} = \{1, 2, \dots, K\}$. The SUs opportunistically use the spectrum licensed by the PUs via an Access Point (AP), as illustrated in Fig.1. Assume there are L PUs and each PU registers part of the licensed spectrum, named as a sub-band. The bandwidth is divided into N_t OFDM subchannels in the CR network. Throughout this paper, we assume that the CR AP has perfect knowledge of Channel State Information (CSI) between the transmitter of the AP and the receivers of the SUs, as well as perfect CSI between the transmitter of the AP and the receivers of the active PUs. As a result, the sum capacity of the CR system obtained by our proposed algorithms will serve as an upper bound of the achievable capacity with channel estimation errors.

Let $\mathcal{M}^l$ denote the set of subchannels corresponding to the sub-band licensed to the lth PU. With periodic spectrum sensing, the CR network identifies non-active frequency bands and selects a subset $\mathcal{N} = \{1, \dots, N\}$ among the subchannels to transmit information. In other words, only the subchannels in vacant sub-bands can be used by the CR network. There are K_0 NRT users with proportional rate constraints, and $K - K_0$ RT users with fixed rate requirements R_k^{req}. The bandwidth of each subchannel is B and the nominal spectrum of the nth subchannel spans from $f_s + (n-1)B$ to $f_s + nB$, where f_s is the starting frequency. When the CR system transmits information over the nth subchannel with unit transmission power, the interference introduced to the jth subchannel in the sub-band of the lth PU is [28]

$$I_{j,l}^n = \int_{(j-1)B-(n-1/2)B}^{jB-(n-1/2)B} g_{n,l}\phi(f)df, \tag{1}$$

TABLE I
LIST OF TERMINOLOGIES AND SYMBOL NOTATIONS

B	The bandwidth of each subchannel
$c_{k,n}$	The channel gain between the AP and the receiver of the kth SU over the nth subchannel
$g_{n,l}$	The power gain from the AP to the receiver of the lth PU over the nth subchannel
$H_{k,n}$	The SNR of the nth subchannel used by the k SU with unit power
$\mathcal{H}_n$ ($\widetilde{\mathcal{H}}_n$)	The hypotheses of the presence (absence) of the PUs' signal on the nth subchannel
$I^n_{j,l}$	The interference to the jth subchannel in the sub-band of the lth PU when an SU transmitting over the nth subchannel with unit power
$I_{n,l}$	The interference to the lth PU by an SU's access on the nth subchannel with unit power
I_l^{th}	The interference threshold for the lth PU
I_k	The interference to the kth SU caused by the PUs' signals
$\mathcal{K}$	The set of SUs
K	The number of SUs
K_0	The number of NRT users
L	The number of PUs
M^l	The set of subchannels corresponding to the sub-band licensed to PU l
$M_v^l(M_o^l)$	The set of subchannels available (unavailable) in the sub-band of the lth PU
$\mathcal{N}$	The set of vacant subchannels determined by the AP
N	The number of vacant subchannels determined by the AP
N_t	The number of subchannels
$\mathcal{O}_n$ ($\widetilde{\mathcal{O}}_n$)	The events that the nth subchannel is unavailable or available determined by the sensing result
$p_{k,n}$	The power allocated on the nth subchannel used by the kth SU
$p^M_{k,n}$	The maximum possible power allocated to the nth subchannel used by the kth SU
$P_{1,n}(P_{2,n})$	The probability that the nth subchannel is truly used by a PU while the CR network makes a correct (incorrect) judgement
P_T	The transmission power limit of the CR network
q_n^L	The priori probability that the sub-band of the nth subchannel is used by a PU
q_n^m (q_n^f)	The probabilities of misdetection (false alarm)
$r_{k,n}$	The rate on the nth subchannel used by the kth SU
$r^M_{k,n}$	The highest achievable rate of the nth subchannel used by the kth SU
R_k	The rate of the kth SU
R_k^{req}	The rate requirement of the kth SU for RT users
$\phi(f)$	The power spectrum density (PSD) of OFDM signal
$\rho_{k,n}$	The subchannel allocation index
γ_k	The rate proportionality factor of the kth SU for NRT users

where $g_{n,l}$ is the power gain from the AP to the receiver of the lth PU on the nth subchannel. $\phi(f) = T(\frac{\sin \pi f T}{\pi f T})^2$ is the Power Spectrum Density (PSD) of OFDM signal, T is OFDM symbol duration.

In practical systems, there are typically two kinds of sensing errors [25]. The first is *misdetection*, which occurs when the CR system fails to detect the PUs' signals. The band of a subchannel is identified to be vacant but it is truly used by the PU. The other kind of sensing errors is *false alarm*, which means the CR network identifies the band of a subchannel is unavailable but it is vacant actually. Generally, the AP in the CR system collects the sensed information of all SUs and makes a decision on which subchannel can be used by SUs. Then the set of sensed available subchannels $\mathcal{M}_v^l$ in the sub-band of the lth PU is determined, as well as the set of sensed unavailable subchannels $\mathcal{M}_o^l$. The probabilities of misdetection and false alarm on the nth subchannel are q_n^m and q_n^f, respectively. Again, we assume the values of q_n^m and q_n^f can be obtained by either local spectrum sensing via the transceivers of SUs, or preferably, cooperative spectrum sensing [25] and distributed spectrum clustering [29]. Note that we do not consider the coexistence of both PUs' and SUs' transmitters in this work, which may arise in practical CR systems. For this case, interference probability is a useful metric and collaborative sensing scheme for the presence of multiple cochannel transmitters is necessary [30].

Obviously, misdetection results in co-channel interference to the PUs, while false alarm lowers the utilization efficiency of spectrum. There are four possible scenarios for spectrum sensing, as shown in the Table II, where $\mathcal{H}_n$ and $\mathcal{O}_n$ are the hypotheses of the absence and presence of a certain PU's signal on the nth subchannel, $\widetilde{\mathcal{H}}_n$ and $\widetilde{\mathcal{O}}_n$ are the events that the nth subchannel is available or unavailable based on the sensed information, respectively. Denote $P_{1,n}$ as the probability that the nth subchannel is truly used by a PU while the CR network makes a correct judgement, we have

$$\begin{aligned} P_{1,n} &= P\{\mathcal{O}_n|\widetilde{\mathcal{O}}_n\} \\ &= \frac{P\{\widetilde{\mathcal{O}}_n|\mathcal{O}_n\}P\{\mathcal{O}_n\}}{P\{\widetilde{\mathcal{O}}_n|\mathcal{O}_n\}P\{\mathcal{O}_n\} + P\{\widetilde{\mathcal{O}}_n|\mathcal{H}_n\}P\{\mathcal{H}_n\}} \\ &= \frac{(1-q_n^m)q_n^L}{(1-q_n^m)q_n^L + q_n^f(1-q_n^L)}, \end{aligned} \tag{2}$$

where q_n^L is the priori probability that the sub-band of the nth subchannel is used by PUs. Similarly, let $P_{2,j}$ denote the probability that the jth subchannel is truly occupied when the CR system deems it as vacant.

Then the interference introduced to the lth PU by the access of an SU on the nth subchannel with unit transmission power is

$$I_{n,l} = \sum_{j\in\mathcal{M}_o^l} P_{1,j} I^n_{j,l} + \sum_{j\in\mathcal{M}_v^l} P_{2,j} I^n_{j,l}. \tag{3}$$

TABLE II
PROBABILITY INFORMATION FROM IMPERFECT SPECTRUM SENSING

	Actual state	Sensing result	Probability information
1	Active ($\mathcal{O}_n$)	Occupied ($\widetilde{\mathcal{O}}_n$)	$P\{\widetilde{\mathcal{O}}_n \vert \mathcal{O}_n\} = 1 - q_n^m$
2	Active ($\mathcal{O}_n$)	Vacant ($\widetilde{\mathcal{H}}_n$)	$P\{\widetilde{\mathcal{H}}_n \vert \mathcal{O}_n\} = q_n^m$
3	Idle ($\mathcal{H}_n$)	Vacant ($\widetilde{\mathcal{H}}_n$)	$P\{\widetilde{\mathcal{H}}_n \vert \mathcal{H}_n\} = 1 - q_n^f$
4	Idle ($\mathcal{H}_n$)	Occupied ($\widetilde{\mathcal{O}}_n$)	$P\{\widetilde{\mathcal{O}}_n \vert \mathcal{H}_n\} = q_n^f$

The transmission rate on the nth subchannel used by the kth SU is

$$r_{k,n} = \log\left(1 + \frac{p_{k,n}|c_{k,n}|^2}{\Gamma(BN_0 + I_k)}\right) \tag{4}$$

where $p_{k,n}$ is the power allocated to the nth subchannel, $c_{k,n}$ is the channel gain between the AP and the receiver of the kth SU over the nth subchannel, N_0 is the PSD of additive white Gaussian noise and Γ is the SNR gap. For an uncoded MQAM, Γ is related to a given BER and $\Gamma = -\ln(5\text{BER})/1.5$ [31]. The interference caused by the PUs' signals to the kth SU is I_k. I_k can be regarded as noise and measured by the receivers of SUs [32], or calculated by the proposed method in [12]. Since cognitive radio system is receiver centric [32], the former is more practical in wireless systems.

Denote $H_{k,n} = \dfrac{|c_{k,n}|^2}{\Gamma(BN_0 + I_k)}$, the rate of the kth user can be expressed as

$$R_k = \sum_{n=1}^{N} \rho_{k,n} \log(1 + p_{k,n} H_{k,n}), \tag{5}$$

where $\rho_{k,n}$ can either be 1 or 0 informing whether the subchannel n is occupied by the kth SU or not. We try to maximize the downlink sum capacity of the AP while guaranteeing the rate requirements of all SUs, under power limitation and interference constraints, the optimization problem is following,

$$\begin{aligned}
\max_{\rho_{k,n}, p_{k,n}} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{k,n} \log(1 + p_{k,n} H_{k,n}) \\
s.t.\ C1 \quad & p_{k,n} \geq 0, \forall n \in \mathcal{N}, \forall k, \\
C2 \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{k,n} p_{k,n} \leq P_T, \\
C3 \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{k,n} p_{k,n} I_{n,l} \leq I_l^{th}, l = 1, \ldots, L \\
C4 \quad & R_1 : R_2 : \ldots : R_{K_0} = \gamma_1 : \gamma_2 : \ldots : \gamma_{K_0}, \\
C5 \quad & R_k = R_k^{req}, k = K_0 + 1, \ldots, K, \\
C6 \quad & \rho_{k,n} \in \{0, 1\}, \forall n \in \mathcal{N}, \forall k, \\
C7 \quad & \sum_{k=1}^{K} \rho_{k,n} = 1, \forall n \in \mathcal{N},
\end{aligned} \tag{6}$$

where P_T is the transmission power limit of the AP. C3 means that the interference to the lth PU can not exceed its threshold I_l^{th}. C4 is the proportional rate constraints of the NRT users. C5 is the fixed rate requirements of the RT users. C6 and C7 indicate that each subchannel is not shared by SUs.

III. INTEGER SUBCHANNEL ALLOCATION

Note that the (6) defines a mixed integer programming problem that involves both binary variables $\rho_{k,n}$'s and real variables $p_{k,n}$'s for optimization. It is generally intractable. Furthermore, the objective function in the (6) is not jointly-convex for $\{\rho_{k,n}, p_{k,n}\}$, hence simply relaxing the integer constraint of $\rho_{k,n}$'s as real still suffers from non-convex problem, which makes it more difficult to solve. One possible approach in dealing with non-convex mixed integer programming problems is to apply the minimax convex relaxation technique [33]. However, it still needs to solve a series of convex optimization problems which is not so efficient when the number of variables becomes large, which is always the case of our considered problem.

To reduce complexity and make the problem tractable, we consider a two-stage approach which has achieved success in diverse scenarios [34, 35]. Specifically, we separate the RA into two individual procedures, subchannel allocation and power distribution. In this section, we first propose an efficient subchannel allocation scheme, which removes the integer constraints in (6) while roughly satisfying the rate constraints of the SUs.

In an OFDM-based CR network, the subchannel with the higher SNR may also generate more interference to the PUs if the sub-band of this subchannel is adjacent to the band used by the PUs, which limits the maximum possible power over this subchannel. Hence, the integer-tone assignment procedure should consider the SNR of a subchannel and the interference to the PUs. In other words, the possible maximum rate of a subchannel is bounded by the SNR and the interference level. Denote $r_{k,n}^M$ as the highest achievable rate of the nth subchannel used by the kth SU, we have

$$r_{k,n}^M = \log(1 + p_{k,n}^M H_{k,n}), \tag{7}$$

where $p_{k,n}^M$ is the maximum possible power allocated to the nth subchannel,

$$p_{k,n}^M = \min(P_T, \min_{l \in \mathcal{L}} I_l^{th}/I_{n,l}). \tag{8}$$

The SNR of a subchannel and the interference to the PUs are linked to a unified capacity in this way. The normalized maximum rate is a practicable criterion to measure the QoS of a subchannel [13], giving insightful hints for subchannel allocation.

Since there are RT users and NRT users in our considered CR network, we separate the subchannel allocation procedure in two steps. First, the RT users have the priority to select subchannels until the rate requirements of all RT users are satisfied. Then the remaining subchannels are allocated to the NRT users based on their proportional rate constraints. The outline of our proposed subchannel allocation scheme is described in Table III, where Ω_k denotes the set of subchannels allocated to the kth user.

At the first round, the RT user whose rate is the farthest away from its target rate has the priority to get a new subchannel among the available ones. Preferably, the subchannel with the highest achievable rate associated with this user will be chosen and the power distribution is temporarily set

TABLE III
SUBCHANNEL ALLOCATION

Algorithm 1
1.**Initialization**:
2.Current rate for each SU: $\mathcal{R}=\{R_1,\ldots,R_K\}=\mathbf{0}$
3.$\mathcal{N}^t=\mathcal{N}$, $\Omega_k=\emptyset,\forall k$
4.**First round for RT users**
5.***While*** $\mathcal{N}_t\neq\emptyset$ and $\min(R_k-R_k^{req})<0$
6. Find k^* satisfies $R_{k^*}/R_{k^*}^{req}\leq R_k/R_k^{req}$, for $k\in$ RT users
7. For k^*, find n^* satisfies $r_{k^*,n^*}^M\geq r_{k^*,n}^M,\forall n\in\mathcal{N}_t$
8. $\mathcal{N}_t=\mathcal{N}_t/n^*$, $\Omega_{k^*}=\Omega_{k^*}\cup n^*$;
9. $R_{k^*}=R_{k^*}+\log(1+p_{k^*,n^*}H_{k^*,n^*})$;
10.***Endwhile***
11.**Second round for NRT users**
12.***While*** $\mathcal{N}_t\neq\emptyset$;
13. Find k^* satisfies $R_{k^*}/\gamma_{k^*}\leq R_k/\gamma_k$, for $k\in$ NRT users;
14. For k^*, find n^* satisfies $r_{k^*,n^*}^M\geq r_{k^*,n}^M,\forall n\in\mathcal{N}_t$;
15. $\mathcal{N}_t=\mathcal{N}_t/n^*$, $\Omega_{k^*}=\Omega_{k^*}\cup n^*$;
16. $R_{k^*}=R_{k^*}+r_{k^*,n^*}^M$;
17.***Endwhile***

as $p_{k,n}=\min(P_T/N,\min\limits_{l\in\mathcal{L}}I_l^{th}/I_{n,l})$. Then the NRT user who suffers the severest unjustness, is given a privilege to choose a subchannel with the highest achievable rate from the remaining ones. The procedure terminates until all subchannels are consumed. Either the fixed rate requirements or the proportional rate constraints are roughly satisfied with our proposed subchannel allocation scheme. The exact satisfaction of the rate constraints will be ultimately accomplished after power distribution among subchannels.

IV. EFFICIENT BARRIER METHOD FOR OPTIMAL POWER ALLOCATION

Given a subchannel assignment, the binary variables $\rho_{k,n}$'s in the (6) are fixed to 0 or 1, the integer constraints vanish and power distribution across subchannels follows. Recall that Ω_k is the set of subchannels allocated to the kth SU, the power distribution problem can be written as follows,

$$\begin{aligned}\max_{r_{k,n}}\quad & \sum_{k=1}^{K}\sum_{n\in\Omega_k} r_{k,n},\\ s.t.\ C1\quad & p_{k,n}\geq 0,\forall n\in\mathcal{N},\forall k,\\ C2\quad & \sum_{k=1}^{K}\sum_{n\in\Omega_k} p_{k,n}\leq P_T,\\ C3\quad & \sum_{k=1}^{K}\sum_{n\in\Omega_k} p_{k,n}I_{n,l}\leq I_l^{th}, l=1,\ldots,L\\ C4\quad & \sum_{n\in\Omega_k} r_{k,n}=\beta_k\sum_{n\in\Omega_1} r_{1,n}, k=2,...,K_0,\\ C5\quad & \sum_{n\in\Omega_k} r_{k,n}=R_k^{req}, k=K_0+1,\ldots,K,\end{aligned}\tag{9}$$

where $\beta_k=\gamma_k/\gamma_1, k=1,\ldots,K_0$. It is easy to prove that the (9) defines a convex problem that can be solved by standard convex optimization techniques, such as *barrier* method [36].

A. The Barrier Method

Barrier method is one of the convex optimization algorithms. By introducing a logarithmic barrier function with parameter t, the *barrier* method makes the inequality constraints implicit in the optimization objective and converts the original problem into a sequence of linear equality constrained minimization problems, the solution to which is called a central point in the central path related to the original problem. The central point will more accurately approximate to the optimal solution as the parameter t increases. For searching the center point with a given t, Newton method is generally employed. The computational complexity of the *barrier* method mainly lies in the computation of Newton step which needs matrix inversion with complexity of $O((N+K))^3$ for our considered problem.

First, we convert all inequality constraints into a logarithmic barrier function $\phi(\boldsymbol{r})$,

$$\begin{aligned}\phi(\boldsymbol{r})=\ & -\sum_{k=1}^{K}\sum_{n\in\Omega_k}\log r_{k,n}-\log(P_T-\sum_{k=1}^{K}\sum_{n\in\Omega_k}p_{k,n})\\ & -\sum_{l=1}^{L}\log(I_l^{th}-\sum_{k=1}^{K}\sum_{n\in\Omega_k}p_{k,n}I_{n,l}),\end{aligned}\tag{10}$$

where all variables $r_{k,n}$'s are collected into a unified vector $\boldsymbol{r}$, and $\boldsymbol{r}=\{r_n\}_{n=1}^N$. Notice that the subscript k can be omitted as it is fixed for a given subchannel assignment. Denote $f(\boldsymbol{r})=\sum\limits_{k=1}^{K}\sum\limits_{n\in\Omega_k}r_{k,n}$, the minimization problem with a certain parameter t is

$$\begin{aligned}\min\quad & \psi_t(\boldsymbol{r})=-tf(\boldsymbol{r})+\phi(\boldsymbol{r})\\ s.t.\quad & A\boldsymbol{r}=b,\end{aligned}\tag{11}$$

where A is a $(K-1)\times N$ matrix and $b\in\mathcal{R}^{K-1}$ with

$$\begin{aligned}A_{k,n}&=\begin{cases}-\beta_{k+1} & k=1,\ldots,K_0-1,n\in\Omega_1\\ 1 & k=2,\ldots,K-1,n\in\Omega_k\\ 0 & otherwise.\end{cases},\\ b_k&=\begin{cases}0 & k=1,\ldots,K_0-1\\ R_k^{req} & k=K_0,\ldots,K-1\end{cases}.\end{aligned}\tag{12}$$

The optimal solution to the (11) is an approximation of the original problem. As t increases, the approximation becomes more and more close to the optimal solution.

At the centering step of the *barrier* method, Newton method is employed to compute the central point. With a given parameter t, Newton step $\Delta\boldsymbol{r}$ and the associated dual variables ν are given by the following Karush-Kuhn-Tucker (KKT) systems,

$$\begin{bmatrix}\bigtriangledown^2\psi_t(\boldsymbol{r}) & A^T\\ A & \mathbf{0}_n\end{bmatrix}\begin{bmatrix}\Delta\boldsymbol{r}\\ \nu\end{bmatrix}=\begin{bmatrix}-\bigtriangledown\psi_t(\boldsymbol{r})\\ \mathbf{0}_v\end{bmatrix},\tag{13}$$

where $\mathbf{0}_n\in\mathcal{R}^{(K-1)\times(K-1)}$ and vector $\mathbf{0}_v\in\mathcal{R}^{(K-1)\times 1}$. $\bigtriangledown^2\psi_t(\boldsymbol{r})$ and $\bigtriangledown\psi_t(\boldsymbol{r})$ are the Hessian and the gradient of $\psi_t(\boldsymbol{r})$, respectively.

The outline of the *barrier* method is summarized in Table IV. ϵ and ϵ_n are the tolerances of the *barrier* method and the Newton step, respectively. α and β are two constants utilized in backtracking line search with $\alpha\in(0,0.5)$ and $\beta\in(0,1)$. The step size of the backtracking line search is s with $s>0$. t and μ is a parameter that is associated with a tradeoff between outer iterations and inner iterations.

TABLE IV
THE BARRIER METHOD

Algorithm 2
1.**Initialization for the Barrier method**
2. Find strictly feasible point $\boldsymbol{r}$, $t := t^{(0)} > 0$, tolerance $\epsilon > 0, \mu > 1$
3.**Outer Loop for Barrier method**
4. Centering step: Compute $\boldsymbol{r}^*(t)$ derived by problem (11)
5. **Initialization for Newton method**
6. Starting point $\boldsymbol{r}$, subject to $A\boldsymbol{r} = b$,
7. tolerance $\epsilon_n > 0, \alpha \in (0, 1/2), \beta \in (0, 1)$
8. **Inner Loop for Newton method**
9. Compute $\Delta\boldsymbol{r}$ and $\lambda := -\bigtriangledown \psi_t(\boldsymbol{r})\Delta\boldsymbol{r}$;
10. Quit if $\lambda^2/2 \leq \epsilon_n$
11. Backtracking line search on $\psi_t(\boldsymbol{r})$, $s := 1$
12. ***while*** $\psi_t(\boldsymbol{r} + s\Delta\boldsymbol{r}) > \psi_t(\boldsymbol{r}) - \alpha s \lambda^2$
13. $s := \beta s$
14. ***endwhile***
15. Update:$\boldsymbol{r} := \boldsymbol{r} + s\Delta\boldsymbol{r}$
16. Update:$\boldsymbol{r}^*(t) = \boldsymbol{r}$.
17. Stopping criterion:$(N+2)/t < \epsilon$.
18. Increase:$t := \mu t$.

B. Speedup the Newton Step

If computing the Newton step by solving the (13) directly, it generates a complexity of $O((N+K)^3)$, which is too high to apply in practical systems. We propose a fast computation of the Newton step by exploiting the structure of the (9). Denote

$$\begin{aligned} f_0 &= P_T - \sum_{k=1}^{K} \sum_{n\in\Omega_k} p_{k,n}, \\ f_l &= I_l^{th} - \sum_{k=1}^{K} \sum_{n\in\Omega_k} p_{k,n} I_{n,l}, l = 1, \ldots, L, \end{aligned} \tag{14}$$

the gradient and the Hessian of $\psi_t(\boldsymbol{r})$ are given by

$$\bigtriangledown \psi_t(\boldsymbol{r}) = -t - \frac{1}{r_n} + \frac{e^{r_n}}{f_0 H_n} + \sum_{l=1}^{L} \frac{I_{n,l} e^{r_n}}{f_l H_n}, \tag{15}$$

$$\begin{aligned} \bigtriangledown^2 \psi_t(\boldsymbol{r}) &= \begin{bmatrix} 1/r_1^2 & & \\ & \ddots & \\ & & 1/r_N^2 \end{bmatrix} + \frac{\bigtriangledown f_0 \bigtriangledown f_0^T}{f_0^2} \\ & \quad + \frac{\bigtriangledown^2 f_0}{f_0} + \sum_{l=1}^{L} \frac{\bigtriangledown f_l \bigtriangledown f_l^T}{f_l^2} + \sum_{l=1}^{L} \frac{\bigtriangledown^2 f_l}{f_l} \\ &= \begin{bmatrix} D_1 & & \\ & \ddots & \\ & & D_N \end{bmatrix} + g_0 g_0^T + \sum_{l=1}^{L} g_l g_l^T. \end{aligned} \tag{16}$$

where $D_n = 1/r_n^2 + e^{r_n}/(H_n f_0) + \sum_{l=1}^{L} e^{r_n} I_{n,l}/(H_n f_l), n \in \mathcal{N}$ and $g_i = \bigtriangledown f_i / f_i, i = 0, 1, \ldots, L$.

The Hessian is positive definite because the diagonal matrix $D = diag(D_1, \ldots, D_N)$, $g_0 g_0^T$ and $g_l g_l^T$ are all positive definite matrixes. Moreover, since A is a full row rank matrix, the KKT matrix at the left-side of the (13) is invertible. However, if we compute the inversion of the KKT matrix directly, it has a complexity of $O((N+K)^3)$, which is too high to apply because there are thousands of OFDM subchannels in practical wireless systems. By exploiting the structure of the problem, we have the following theorem:

***Theorem** 1:* The Eq.(13) can be solved with a complexity of $O(L^2 N)$.

The detail of the proof is presented in Appendix. We give the sketch here. As can be observed from the (16), the Hessian $\bigtriangledown^2 \psi_t(\boldsymbol{r})$ is a sum of diagonal matrix D and $L+1$ rank-one matrices $g_l g_l^T$'s, which leads to a fast computation of matrix inversion of $\bigtriangledown^2 \psi_t(\boldsymbol{r})$ [37]. Thus, the inversion of the KKT matrix can be computed in a recursive manner with a complexity of $O(L^2 N)$, as illustrated in Appendix.

Based on ***Theorem 1***, we can see the computational cost is significantly reduced by our proposed algorithm compared to matrix inversion with a complexity of $O((N+K)^3)$.

C. Warm Start

In the initialization step of the *barrier* method, it requires a strictly feasible starting point which obeys all the constraints. A preparatory procedure for the *barrier* method to prove and compute the existed feasible point is necessary. As discussed in [36], it is equivalent to solve a minimization problem by introducing a crucial indicator parameter z. So we form an optimization problem to find a strictly feasible solution to (9), which has the following form,

$$\begin{aligned} \min_{z, r_{k,n}} \quad & e^z, \\ s.t.\ C1 \quad & r_{k,n} \geq 0, \forall n \in \mathcal{N}, \forall k, \\ C2 \quad & \sum_{k=1}^{K} \sum_{n\in\Omega_k} p_{k,n} \leq P_T + z, \\ C3 \quad & \sum_{k=1}^{K} \sum_{n\in\Omega_k} p_{k,n} I_{n,l} \leq I_l^{th} + z, l = 1, \ldots, L \\ C4 \quad & \sum_{n\in\Omega_k} r_{k,n} = \beta_k \sum_{n\in\Omega_1} r_{1,n}, k = 2, ..., K_0, \\ C5 \quad & \sum_{n\in\Omega_k} r_{k,n} = R_k^{req}, k = K_0 + 1, \ldots, K, \end{aligned} \tag{17}$$

The variable z can be interpreted as an upper bound of the maximum infeasibility of the inequalities, as can be seen from the constraints C2 of the (17). If the optimal solution of the (17) is less than or equal to zero, it means that at least one feasible point exists and we can take it as a warm start to solve the (9). Since the (17) has the similar structure with the (9), the fast *barrier* method discussed above can also be applied to solve the (17). Obviously, it is easy to give a feasible starting point for z and $r_{k,n}$'s to solve the (17). Note that the optimal solution to the (17) can be greater than zero. If this case happens, no feasible solution to the (9) exists and we regard this case as system outage.

V. A SUBOPTIMAL POWER ALLOCATION ALGORITHM

In this section, we propose a constant complexity power allocation algorithm to approach the optimal solution. Inspired by the index function concept proposed in [13], we introduce a normalized cost function to measure the cost of allocating a given rate over each subchannel,

$$F_c(r_{k,n}) = \frac{e^{r_{k,n}} - 1}{e^{r_{k,n}^M} - 1}. \tag{18}$$

The SNR of a subchannel and the interference generated to the PUs are jointly considered in the normalized cost function. Then we can convert the power allocation problem into the following form,

$$\begin{aligned} \max_{r_{k,n}} \quad & \sum_{k=1}^{K} \sum_{n\in\Omega_k} r_{k,n} \\ s.t.\ C1 \quad & r_{k,n} \geq 0, \forall k, n \in \Omega_k \\ C2 \quad & \sum_{k=1}^{K} \sum_{n\in\Omega_k} F_c(r_{k,n}) \leq C \\ C3 \quad & \sum_{n\in\Omega_k} r_n = \beta_k \sum_{n\in\Omega_1} r_n, k = 2, ..., K_0 \\ C4 \quad & \sum_{n\in\Omega_k} r_n = R_k^{req}, k = K_0+1, \dots, K, \end{aligned} \tag{19}$$

where C in C2 is a constant jointly determined by the transmission power and the interference constraints, informing the maximum sum capacity of the CR system. The solution to the (19) can be regarded as an approximation to the (9). Notice that we do not know the exact value of C in advance. It can be only worked out when all $r_{k,n}$'s are obtained. We will show it is not necessary to know C when solving the (19).

The Lagrangian of the (19) is

$$\begin{aligned} L = \quad & -\sum_{k=1}^{K} \sum_{n\in\Omega_k} r_{k,n} + \lambda(\sum_{k=1}^{K} \sum_{n\in\Omega_k} F_c(r_{k,n}) - C) \\ & + \sum_{k=2}^{K_0} \mu_k(\beta_k \sum_{n\in\Omega_k} r_{k,n} - \sum_{n\in\Omega_k} r_{k,n}) \\ & + \sum_{k=K_0+1}^{K} \mu_k(R_k^{req} - \sum_{n\in\Omega_k} r_{k,n}), \end{aligned} \tag{20}$$

where $\lambda > 0$ and $\{\mu_k\}_{k=2}^{K}$ are the Lagrange multipliers. The KKT conditions of the (20) are following,

$$\begin{aligned} \frac{\partial L}{\partial r_{1,n}} &= -1 + \lambda \frac{e^{r_{1,n}}}{e^{r_{1,n}^M}-1} + \sum_{k=2}^{K_0} \mu_k\beta_k = 0, n \in \Omega_1 \\ \frac{\partial L}{\partial r_{k,n}} &= -1 + \lambda \frac{e^{r_{k,n}}}{e^{r_{k,n}^M}-1} - \mu_k = 0, k \geq 2, n \in \Omega_k. \end{aligned} \tag{21}$$

Redefine $r_{k,n}$ as the rate on the nth subchannel used by the kth SU, the rate distribution for each SU satisfies the following equation,

$$r_{k,n} - r_{k,m} = \log(\frac{p_{k,n}^M H_{k,n}}{p_{k,m}^M H_{k,m}}), k = 1, \dots, K. \tag{22}$$

Without loss generality, assume $r_{k,1}^M \leq r_{k,2}^M \leq \dots \leq r_{k,N_k}^M$ and denote $v_{k,n} = \frac{p_{k,n}^M H_{k,n}}{p_{k,1}^M H_{k,1}}$, we have

$$r_{k,n} = r_{k,1} + \log v_{k,n}, n = 1, \dots, N_k, \tag{23}$$

where N_k is the number of subchannels allocated to the kth SU. Consequently, the rate of the kth user can be calculated as follows,

$$R_k = N_k r_{k,1} + \sum_{n=1}^{N_k} \log v_{k,n}, \tag{24}$$

and the power consumption and interference generated to the lth PU by each SU are given by

$$\begin{aligned} P_k &= \sum_{n=1}^{N_k} e^{r_{k,1}} \frac{v_{k,n}}{H_{k,n}} - \sum_{n=1}^{N_k} \frac{1}{H_{k,n}} = e^{r_{k,1}} X_k^0 - Y_k^0 \\ I_k^l &= \sum_{n=1}^{N_k} e^{r_{k,1}} \frac{v_{k,n} I_{n,l}}{H_{k,n}} - \sum_{n=1}^{N_k} \frac{I_{n,l}}{H_{k,n}} = e^{r_{k,1}} X_k^l - Y_k^l. \end{aligned} \tag{25}$$

As fixed rate requirements are imposed to the RT users, we can obtain the rate allocation for the RT users from the following equation,

$$N_k r_{k,1} + \sum_{n=1}^{N_k} \log v_{k,n} = R_k^{req}. \tag{26}$$

Then other $r_{k,n}$s for user k can be easily derived from the (23).

Denote $P_r = \sum_{k=K_0+1}^{K} P_k$ and $I_r^l = \sum_{k=K_0+1}^{K} I_k^l$ as the power consumed and the interference introduced to the lth PU by the RT users, respectively. For the NRT users, there is a set of proportional rate constraints to be satisfied. From the equation $\sum_{n=1}^{N_k} r_{k,n} = \beta_k \sum_{n=1}^{N_1} r_{1,n}$, we have

$$\begin{aligned} r_{k,1} &= \frac{N_1\beta_k}{N_k} r_{1,1} + \frac{1}{N_k}(\beta_k \sum_{n=1}^{N_1} v_{1,n} - \sum_{n=1}^{N_k} v_{k,n}) \\ &= V_k r_{1,1} + W_k. \end{aligned} \tag{27}$$

Consequently,

$$\begin{aligned} P_k &= e^{V_k r_{1,1} + W_k} X_k^0 - Y_k^0 \\ I_k^l &= e^{V_k r_{1,1} + W_k} X_k^l - Y_k^l, l = 1, \dots, L. \end{aligned} \tag{28}$$

According to the transmission power limit and the interference threshold, the rate $r_{1,1}$ is constrained by the following inequalities,

$$\begin{aligned} & \sum_{k=1}^{K_0} e^{V_k r_{1,1} + W_k} X_k^0 - Y_k^0 \leq P_T - P_r, \\ & \sum_{k=1}^{K_0} e^{V_k r_{1,1} + W_k} X_k^l - Y_k^l \leq I_l^{th} - I_r^l, l = 1, \dots, L. \end{aligned} \tag{29}$$

The maximum value of $r_{1,1}$ can be worked out by using bisection method. And other $r_{k,n}$'s are obtained by insert $r_{1,1}$ into the (23) and the (27).

However, we also need to consider the constraints that $r_{k,n} \geq 0$ for both the RT and the NRT users. For subchannel n, there is no power allocation if $r_{k,n} < 0$, which may occur when the kth SU does not have a high achievable rate on this subchannel. In this case, we should give up using this subchannel and update the rate allocation.

The computational complexity can be counted roughly as follows. As the rate allocation for the RT users can be obtained from the (23) and the (26) directly, the computational cost mainly lies in the rate allocation for the NRT users. We need to solve $L + 1$ inequalities in the (29) by using bisection method. The number of iterations is about $L \log_2(1/\epsilon_b)$, where ϵ_b is the error tolerance for the bisection method. Hence the complexity of the proposed suboptimal power allocation algorithm is $O(L \log_2(1/\epsilon_b))$, which is almost a constant for ϵ_b with a given number of PUs.

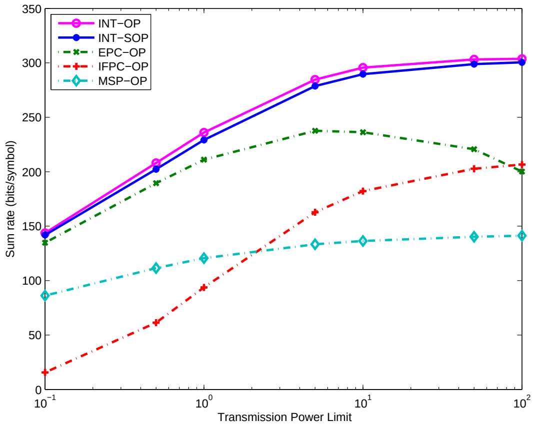

Fig. 2. The sum rate of all SUs as a function of transmission power limit. ($N_t = 64$, $K = 4$, $K_0 = 2$, $L = 2$.)

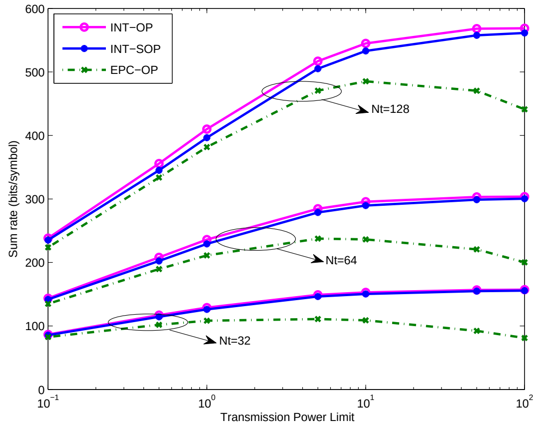

Fig. 3. Sum capacity as a function of transmission power limit. ($K = 4$, $K_0 = 2$,$L = 2$.)

VI. Simulation Results

We conduct a series of experiments to evaluate the performance of the proposed algorithms. Consider a multiuser OFDM-based CR system, where each receiver uniformly distributed in a circle within 0.5km from its transmitter. The path loss exponent is 4, the variance of shadowing effect is 10dB and the amplitude of multipath fading is Rayleigh. The noise power is 10^{-13}W. The probability of PUs' activity (q_n^L), misdetection (q_n^m) and false alarm (q_n^f) are uniform distributed over [0,1], [0.01,0.05] and [0.05,0.1], respectively. The bandwidth of each OFDN subchannel is 62.5kHz. The AP in the CR network identifies available subchannels randomly. We also assume that each PU's bandwidth is randomly generated by uniform distribution among the licensed spectrum.

To evaluate sum capacity, we compare the proposed algorithms, including integer subchannel assignment proposed in Section III with optimal power allocation proposed in Section IV (INT-OP), integer subchannel assignment with rate loading scheme proposed in Section V (INT-SOP), with the other three algorithms: EPC, IFPC and MSP. The EPC and the IFPC are introduced in [24]. The EPC assumes that equal power is distributed among subchannels, while IFPC allocates power inversely proportional to the interference level. The MSP always allocates a subchannel to the user who acquires the highest SNR over this channel. All schemes adopt optimal power allocation proposed in Section IV. There are 64 subchannels, $K = 4$, $K_0 = 2$, $\gamma_1 : \gamma_2 = 1 : 1$ and $R_{k,min} = 10$ bit/symbol. Fig.2 shows the sum capacity as a function of transmission power limit P_T.

From Fig.2 we can observe that the sum capacity of all SUs grows with the increase of power budget. Our proposed algorithms, the INT-OP and the INT-SOP, perform better than the others. When power budget P_T is small, the EPC works quite well. Conversely, the IFPC obtains solutions close to our proposed schemes when P_T is large. The reason is that the power limit and the interference level are jointly considered in our proposed subchannel allocation schemes, while the EPC and the IFPC take only one of them into consideration. It is worthy noticing that the gap between the INT-OP and the INT-SOP is small, suggesting the suboptimal power allocation algorithm proposed in Section V provides a good approximation to the optimal.

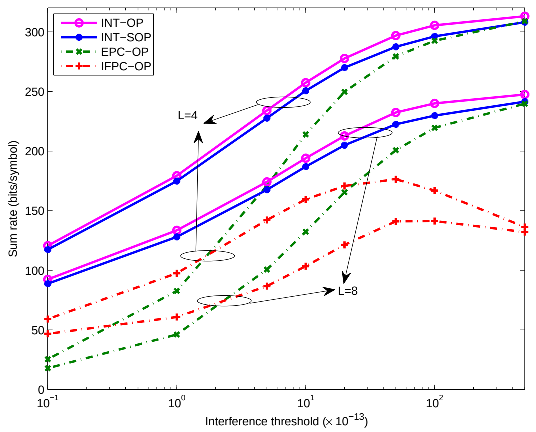

Fig. 4. Sum capacity as a function of the Interference threshold. ($N_t = 64$, $K = 4$, $K_0 = 2$.)

Fig.3 illustrates the sum rate of all SUs versus the transmission power limit for different numbers of subchannels. There are 4 SUs including 2 RT users, whose rate requirements are uniformly set to 20bits/symbol. The interference threshold for each PU is 5×10^{-13}W. It is obvious that the sum rate increases when the number of subchannels becomes larger. We can clarify this phenomenon as a result of the channel diversity in wireless environment. Anyway, the INT-SOP is always capable to achieve more than 97% of the INT-OP in the different cases, which means that the performance loss due to the suboptimal power allocation is negligible.

We depict the sum rate of all SUs versus the interference threshold for different numbers of PUs in Fig.4. There are 64 subcarriers in the considered network and the power limit is 1W. There are 2 NRT users with $\gamma_1 : \gamma_2 = 1 : 1$ and 2 RT users with $R_k^{req} = 10$bits/symbol. All PUs have equal interference thresholds. The sum rate increases with the growth of the interference threshold as can be seen from Fig.4. The

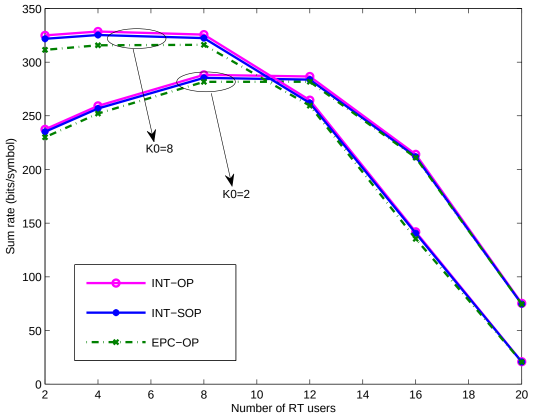

Fig. 5. Sum capacity as a function of number of RT users. ($N_t = 64$, $L = 2$, $R_k^{req} = 20$bit/symbol.)

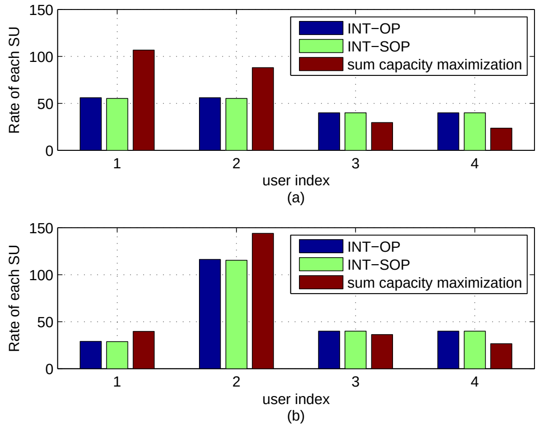

Fig. 6. Normalized rate distribution among SUs. ($K = 4$, $K_0 = 2$, $R_k^{req} =$ 40bit/symbol. (a) $\gamma_1 : \gamma_2 = 1 : 1$; (b) $\gamma_1 : \gamma_2 = 1 : 4$.)

performance of the EPC and IFPC differs with different interference threshold. We can explain this phenomenon intuitively. When the interference threshold is relatively smaller, most of the subchannels are interference limited, which makes the performance of IFPC better than EPC. When the interference threshold increases, more and more subchannels are power limited, and the EPC performs better than IFPC. It is worth noting that the EPC and our proposed algorithms perform well when the interference threshold is large enough. The reason is that all subchannels become power limited on the conditions of large interference tolerance. If the interference threshold is small, more subchannels are interference limited, and the performance gap between the EPC and our proposed methods becomes larger as the decrease of the threshold.

We also verify the effect of multiuser diversity for the CR network. Fig.5 shows the sum capacity versus the number of the RT users for two cases, where the number of the NRT users is $K_0 = 2$ and $K_0 = 8$, respectively. The number of the RT users varies from 2 to 20 with fixed rate requirements of $R_k^{req} = 20$bit/symbol. We assume equal fairness among the NRT users. The number of subchannels is $N_t = 64$ and $P_T = 1W$. When the number of the RT users is relatively small, the sum rate increases slightly with the growth of the number of the RT users. However, the rate loss occurs at a cut-off of the number of RT users, where a sharp decrease of the sum rate can be found when the number of RT users becomes larger. This phenomenon can be explained intuitively.

At the beginning, the CR network benefits from multiuser diversity because a subchannel is more likely to be allocated to an SU that has good channel gain over it. So we can find the sum capacity of the CR network increases with the growth of the number of the RT users. However, when there are more RT users trying to access the CR network, most of the subchannels and more power are consumed by the RT users to meet their rate requirements. And the radio resource will be more frequently exhausted when the number of the RT users is lager than the cut-off, which leads to capacity loss of the NRT users. So does the CR network. We can also see from Fig.5 that the INT-SOP perform almost as well as the INT-OP,

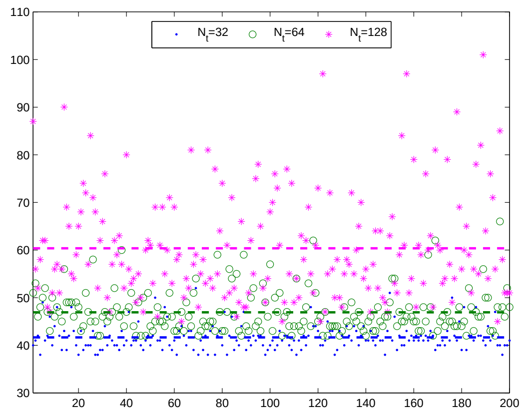

Fig. 7. Number of Newton iterations required for convergence during 200 channel realizations. ($K = 4$, $K_0 = 2$.)

both of which is slightly better than the EPC-OP.

The performance of the fixed rate requirements of the RT users and the proportional rate fairness among the NRT users are shown in Fig.6(a) and Fig.6(b), respectively. We compare our proposed two schemes with sum capacity maximization scheme, which maximizes the sum capacity without any rate constraint. $R_k^{req} = 40$bit/symbol, $\gamma_1 : \gamma_2 = 1 : 1$ and $\gamma_1 : \gamma_2 = 1 : 4$ in Fig.6(a) and Fig.6(b), respectively. $K = 4$ and $K_0 = 2$. The users with index 3 and 4 in Fig.6 are the RT ones. It is shown that our proposed schemes can strictly satisfy the various rate requirements, which is not the case for the sum capacity maximization method.

Finally, we investigate the convergence of our proposed *barrier* method. As discussed in Section IV, the computational load of the *barrier* method mainly lies in the computation of Newton step. If the number of Newton iterations is large or varies in a wide range, the algorithm would be difficult to be applied in practical wireless systems. Fig.7 and Fig.8 show that it is not the case for our proposed method for all concerned settings. Fig.7 shows the number of Newton iterations for the *barrier* method to converge in 200 random instances. The

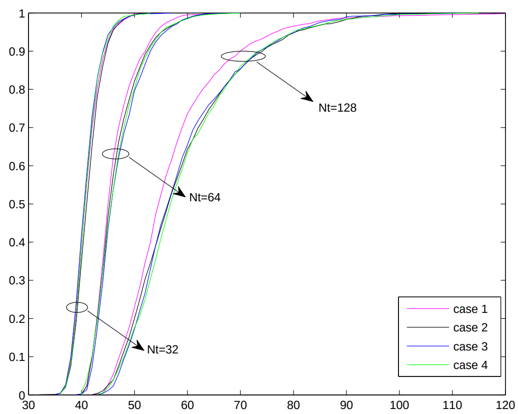

Fig. 8. CDF of the number of Newton iterations required for convergence over 1000 channel realizations. ($K = 4$, $K_0 = 2$, $P_T = 1$W. $\gamma_1 : \gamma_2 = 1 : 1$ (case 1), $\gamma_1 : \gamma_2 = 1 : 2$ (case 2), $\gamma_1 : \gamma_2 = 1 : 3$ (case 3), $\gamma_1 : \gamma_2 = 1 : 4$ (case 4).)

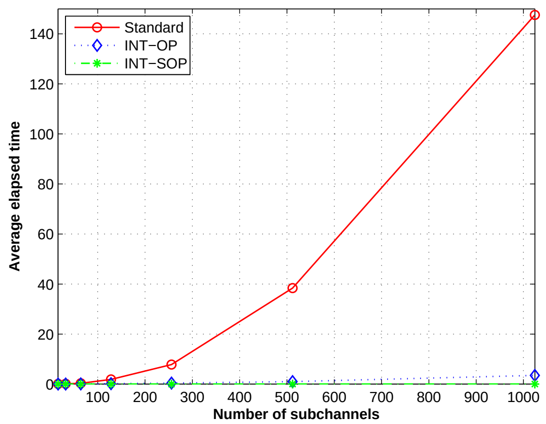

Fig. 9. Average time cost as a function of the number of subchannels.

average number of Newton iterations is shown in Fig.7 by the corresponding dashed line for a certain N_t. Fig.8 gives the cumulative distribution function (CDF) of the number of Newton iterations. Both Fig.7 and Fig.8 show that the number of Newton iterations varies in a narrow range with a given N_t. We also give the time cost of our proposed algorithms (INT-OP and INT-SOP) and the standard one (Standard) which computes Newton step by matrix inversion. Fig.9 shows the average time cost as a function of number of subchannels over 1000 instances. The elapsed time is counted by in-built *tic-tac* function in *Matlab*. From Fig.9 we can see the time cost of our proposed algorithms is much less than the standard technique. Especially for the INT-SOP, the time cost is less than 1ms even the number of subchannels is 1024.

VII. Conclusion

In this paper, we studied the adaptive RA problem in multiuser OFDM-based CR networks with heterogeneous services and imperfect spectrum sensing, which extended our preliminary research [38]. Since we consider many practical limitations, our general system model results that the formulated optimization problem is computationally intractable. To make it tractable, we proposed to address the problem in a two-step style. In the first step, we perform subchannel allocation, by considering different requirements of real-time and non-real-time users. Then, in the second step, we execute power allocation among subchannels. We developed a fast barrier-based method which can achieve the optimal solution with an almost linear complexity, much better than standard techniques, by exploiting problem structure to speedup the Newton step. Furthermore, we also proposed a method which can achieve nearly optimal solution with a constant complexity. Numerical simulations validate the effectiveness and efficiency of our proposed methods. We believe that our proposals throw some insightful light on how to address the resource allocation problem in CR systems. In future work, the case of imperfect channel state information, the presence of multiple users using the same subchannels and the interference among multiple primary cells should be investigated, to make the proposed RA algorithms more promising for applications.

Appendix

Proof of Theorem 1: Rewrite the KKT system (13) into the following form,

$$\Lambda_0 \tilde{x} = G_0, \tag{30}$$

where $\tilde{x} = \begin{bmatrix} \Delta \mathbf{r} \\ \nu \end{bmatrix}$ and $G_0 = \begin{bmatrix} -\bigtriangledown \Psi_t \\ \mathbf{0}_v \end{bmatrix}$. According to the decomposition in (16), Λ_0 can be written into

$$\Lambda_0 = \begin{bmatrix} D & A^T \\ A & \mathbf{0}_n \end{bmatrix} + \sum_{i=1}^{L+1} G_i G_i^T, \tag{31}$$

where $G_i = \begin{bmatrix} g_{i-1} \\ \mathbf{0}_v \end{bmatrix}, i = 1, \ldots, L+1$.

By taking the advantage of the special structure discussed above, we can derive an efficient method to calculate the Newton step. An $L+1$-step procedure for decomposition is required as follows,

Step 1 $\Lambda_0 = \Lambda_1 + G_1 G_1^T$,

where $\Lambda_1 = \begin{bmatrix} D & A^T \\ A & \mathbf{0}_n \end{bmatrix} + \sum_{i=2}^{L+1} G_i G_i^T$.

Particularly, we have $\tilde{x} = \upsilon_1^1 - \frac{G_1 \upsilon_1^1}{1+G_1\upsilon_2^1}\upsilon_2^1$,

where $\Lambda_1 \upsilon_1^1 = G_0$ and $\Lambda_1 \upsilon_2^1 = G_1$.

Step 2 $\Lambda_1 = \Lambda_2 + G_2 G_2^T$,

where $\Lambda_2 = \begin{bmatrix} D & A^T \\ A & \mathbf{0}_n \end{bmatrix} + \sum_{i=3}^{L+1} G_i G_i^T$.

Similarly, $\upsilon_i^1 = \upsilon_i^2 - \frac{G_2 \upsilon_i^2}{1+G_2\upsilon_3^2}\upsilon_3^2, i = 1, 2$,

And $\Lambda_2 \upsilon_i^2 = G_{i-1}, i = 1, 2, 3$.

Without loss of generality, we consider the Step n,

Step *n* Let $\Lambda_{n-1} = \Lambda_n + G_n G_n^T$.

We can update the n variables at Step $n-1$ by

$\upsilon_i^{n-1} = \upsilon_i^n - \frac{G_n^T \upsilon_i^n}{1+G_n\upsilon_{n+1}^n}\upsilon_{n+1}^n, i = 1, \ldots, n$,

where $\Lambda_n \upsilon_i^n = G_{i-1}, i = 1, \ldots, n+1$.

Since we can update the n variables $\upsilon_i^{n-1}, i = 1, \ldots, n$ at Step $n-1$ by the $n+1$ variable $\upsilon_i^n, i = 1, \ldots, n+1$ at Step n, we can figure out the $L+2$ variables $\upsilon_i^{L+1}, i = 1, \ldots, L+2$

at Step $L+1$ with $\Lambda_{L+1}v_i^{L+1} = G_{i-1}$, to calculate the Δx in (30) indirectly.

Now we consider the matrix systems at step $L+1$ in a unified form as follows,

$$\left[\begin{array}{cc} D & A^T \\ A & \mathbf{0}_n \end{array}\right]\left[\begin{array}{c} u \\ v \end{array}\right] = \left[\begin{array}{c} h \\ \mathbf{0}_v, \end{array}\right]. \tag{32}$$

where $u \in \mathcal{R}^{M\times 1}$ and $v \in \mathcal{R}^{(K-1)\times 1}$. And we have

$$\begin{aligned} & D_n u_n + \sum_{k=2}^{K_0} \beta_k v_{k-1} = h_n, n = 1, \dots N_1 \\ & D_{\pi_k+n} u_{\pi_k+n} - v_{k-1} = h_{\pi_k+n}, k \geq 2, n = 1, \dots, N_k \\ & \beta_k \sum_{n=1}^{N_1} u_n - \sum_{n=1}^{N_k} u_{\pi_k+n} = 0, k = 2, \dots, K_0 \\ & \sum_{n=1}^{N_k} u_{\pi_k+n} = 0, k = K_0+1, \dots, K, \end{aligned} \tag{33}$$

where $\pi_k = \sum_{i=1}^{k-1} N_i$. Denote $X_k = \sum_{n=1}^{N_k} u_{\pi_k+n}$, and we have

$$\begin{aligned} X_1 &= \sum_{n=1}^{N_1} \frac{h_n - \sum_{k=2}^{K_0} \beta_k v_{k-1}}{H_n} = B_1 - A_1 \sum_{k=2}^{K_0} \beta_k v_{k-1} \\ X_k &= \sum_{n=1}^{N_k} \frac{h_{\pi_k+n} + v_{k-1}}{H_{\pi_k+n}} = B_k + v_{k-1} A_k, k \geq 2, \end{aligned} \tag{34}$$

where $A_k = \sum_{n=1}^{N_k} \frac{1}{H_{\pi_k+n}}$ and $B_k = \sum_{n=1}^{N_k} \frac{h_{\pi_k+n}}{H_{\pi_k+n}}, k = 1, \dots, K$. Then we have

$$v_{k-1} = \begin{cases} \frac{\beta_k X_1 - B_k}{A_k}, & k = 2, \dots, K_0, \\ -\frac{B_k}{A_k} & k = K_0+1, \dots, K. \end{cases} \tag{35}$$

Substituting the v_k's back into the (34), we have

$$X_1 = (B_1 + A_1 \sum_{k=2}^{K_0} B_k \beta_k / A_k)/(1 + A_1 \sum_{k=2}^{K_0} \beta_k^2 / A_k) \tag{36}$$

Then all variable v_k's can be worked out according to the (35) and the (36). The other variables can also be obtained by substituting v_k's into the (33).

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