Supplemental Material for Active Learning based on Locally Linear Reconstruction

Lijun Zhang, *Student Member, IEEE,* Chun Chen, *Member, IEEE,* Jiajun Bu, *Member, IEEE,* Deng Cai, *Member, IEEE,* Xiaofei He, *Senior Member, IEEE,* Thomas S. Huang, *Life Fellow, IEEE*

Appendix A
Proof of the Equation (22)

Following the Sherman-Morrison-Woodbury formula, we have

$$\begin{aligned}
&(\mu M + \Lambda_n + \Gamma_i)^{-1}\\
=&(\mu M + \Lambda_n + \mathbf{e}_i\mathbf{e}_i^T)^{-1}\\
=&(\mu M + \Lambda_n)^{-1} - \frac{(\mu M + \Lambda_n)^{-1}\mathbf{e}_i\mathbf{e}_i^T(\mu M + \Lambda_n)^{-1}}{1+\mathbf{e}_i^T(\mu M + \Lambda_n)^{-1}\mathbf{e}_i}\\
=&H - \frac{H\mathbf{e}_i\mathbf{e}_i^T H}{1+\mathbf{e}_i^T H\mathbf{e}_i}\\
=&H - \frac{H_{*i}H_{i*}}{1+H_{ii}}
\end{aligned} \tag{1}$$

- L. Zhang, C. Chen and J. Bu are with the Zhejiang Provincial Key Laboratory of Service Robot, College of Computer Science, Zhejiang University, Hangzhou 310027, China.
E-mail: {zljzju,chenc,bjj}@zju.edu.cn
- D. Cai and X. He are with the State Key Lab of CAD&CG, College of Computer Science, Zhejiang University, Hangzhou 310058, China.
E-mail: {dengcai,xiaofeihe}@cad.zju.edu.cn
- T. Huang is with the Beckman Institute for Advanced Sciences and Technology, University of Illinois at Urbana Champaign, 405 North Mathews Ave., Urbana, IL 61801, USA.
E-mail: huang@ifp.uiuc.edu

Appendix B
Proof of the Equation (23)

Since $H_{i*}^T = H_{*i}$, $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$, $\mathrm{Tr}(A^T) = \mathrm{Tr}(A)$ and both M and H are symmetric, we get

$$\begin{aligned}
&\|(\mu M + \Lambda_n + \Gamma_i)^{-1}\mu M X\|_F^2\\
=&\mu^2 \,\mathrm{Tr}\left((\mu M + \Lambda_n + \Gamma_i)^{-1} M X X^T M (\mu M + \Lambda_n + \Gamma_i)^{-1}\right)\\
=&\mu^2 \,\mathrm{Tr}\left((H - \frac{H_{*i}H_{i*}}{1+H_{ii}}) M X X^T M (H - \frac{H_{*i}H_{i*}}{1+H_{ii}})\right)\\
=&\mu^2 \,\mathrm{Tr}\left(H M X X^T M H\right) - \frac{\mu^2 \,\mathrm{Tr}\left(H_{*i}H_{i*} M X X^T M H\right)}{1+H_{ii}}\\
&- \frac{\mu^2 \,\mathrm{Tr}\left(H M X X^T M H_{*i}H_{i*}\right)}{1+H_{ii}}\\
&+ \frac{\mu^2 \,\mathrm{Tr}\left(H_{*i}H_{i*} M X X^T M H_{*i}H_{i*}\right)}{(1+H_{ii})^2}\\
=&\mu^2 \,\mathrm{Tr}\left(H M X X^T M H\right) - \frac{2\mu^2 \,\mathrm{Tr}\left(H_{*i}H_{i*} M X X^T M H\right)}{1+H_{ii}}\\
&+ \frac{\mu^2 H_{i*}H_{*i}H_{i*} M X X^T M H_{*i}}{(1+H_{ii})^2}\\
=&\mu^2 \,\mathrm{Tr}\left(H M X X^T M H\right) - \frac{2\mu^2 H_{i*} M X X^T M H H_{*i}}{1+H_{ii}}\\
&+ \frac{\mu^2 H_{i*}H_{*i}H_{i*} M X X^T M H_{*i}}{(1+H_{ii})^2}
\end{aligned} \tag{2}$$